New Ultrasonic Shear Horizontal (SH) Surface Elastic Waves with Fascinating Properties

Prof. dr hab. inż. Piotr Kiełczyński

Polish Academy of Sciences
Institute of Fundamental Technological Research
Laboratory of Acoustoelectronics, Warsaw, Poland
email: pkielczy@ippt.pan.pl

January 26th 2026, Warsaw, Poland.

Historical perspective for elastic surface waves

Table.1.

No	Year	New Wave	Discovered by	Country	Features
1	1885	Rayleigh waves	Lord Rayleigh	England	Two components of vibrations L, SV, half-space
2	1911	Love waves	A.E.H. Love	England	One SH component of vibrations, layered half-space
3	1917	Lamb waves	H. Lamb	England	Two components of vibrations L, SV, plate
4	1924	Stoneley waves	R. Stoneley	England	Two components of vibrations L, SV, solid-solid interface
5	1947	Scholte waves	J.G. Scholte	Holland	Two components of vibrations L, SV, solid-liquid interface
6	1968-1969	Bleustein-Gulyaev waves	J.L. Bleustein, Yu,V. Gulyaev	USA, Russia	One SH component of vibrations, metalized piezoelectric half-space
7	1971	Maerfeld-Tournois waves	C. Mearfeld and P. Tournois	France	One SH component of vibrations, elastic-piezoelectric interface
8	2022	Kiełczyński waves No.1	P.M. Kiełczyński	Poland	One SH component of vibrations, interface between conventional and metamaterial elastic solid, SPP-like
9	2024	Kiełczyński waves No.2	P.M. Kiełczyński	Poland	One SH component of vibrations, interface between conventional elastic surface layer and metamaterial substrate - Love-like

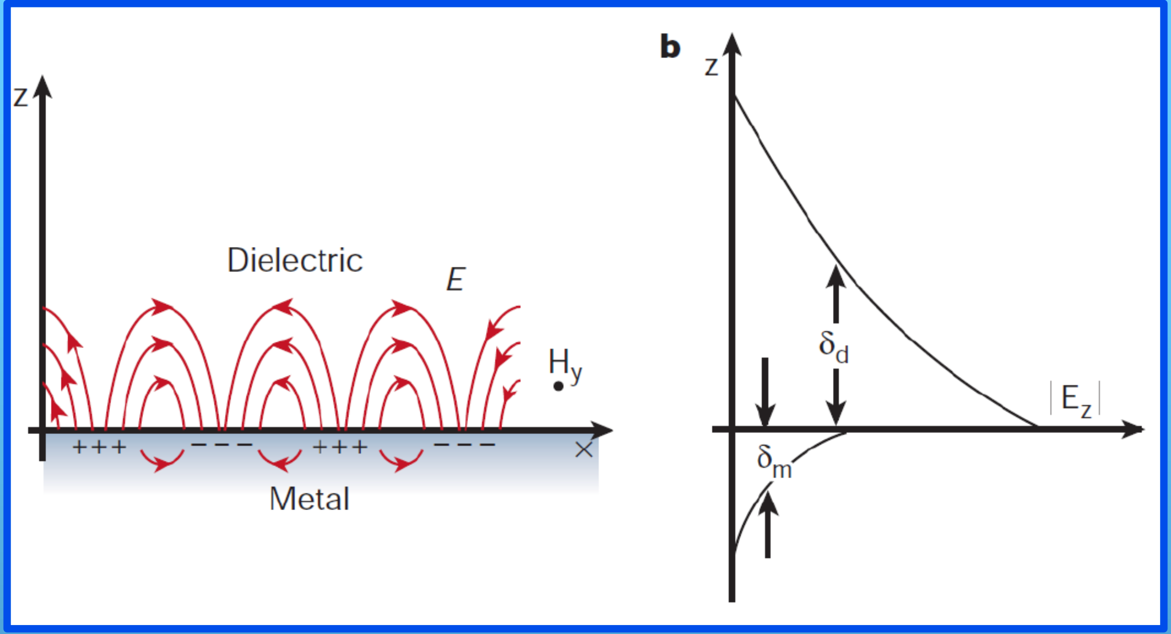


Fig.1.

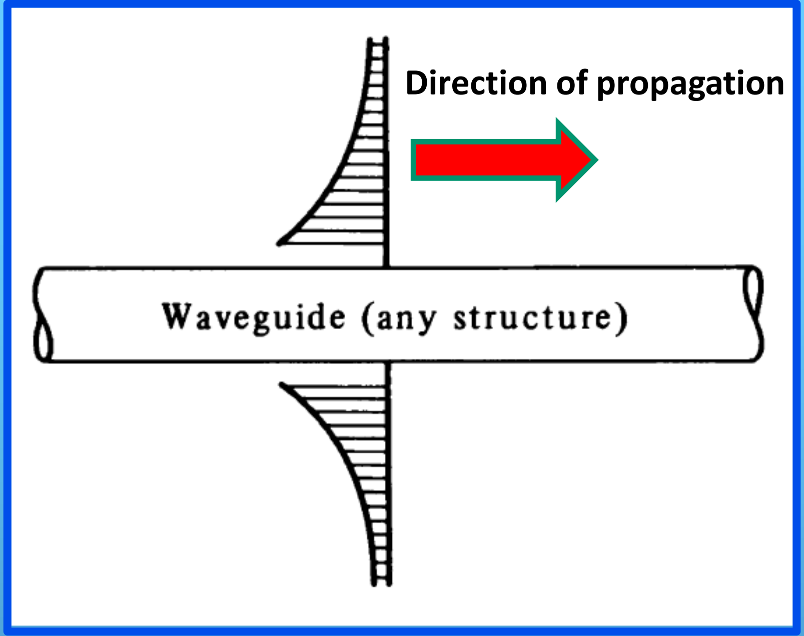


Fig.2.

Surface Plasmon Polariton (SPP) electromagnetic wave at a metal- dielectric interface

$$\varepsilon(\omega) = \varepsilon_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$$

Dielectric constant Plasmonic optical materials

Drude's model of dielectric constant ε in metals:
ω_p = Plasmon angular frequency

Entire presentation is a novelty

The concept of a surface wave:
elastic and/or electromagnetic wave

Newly discovered (SH) Surface Elastic Wave:
Elastic analogue of optical SPP waves

PHYSICAL MODEL

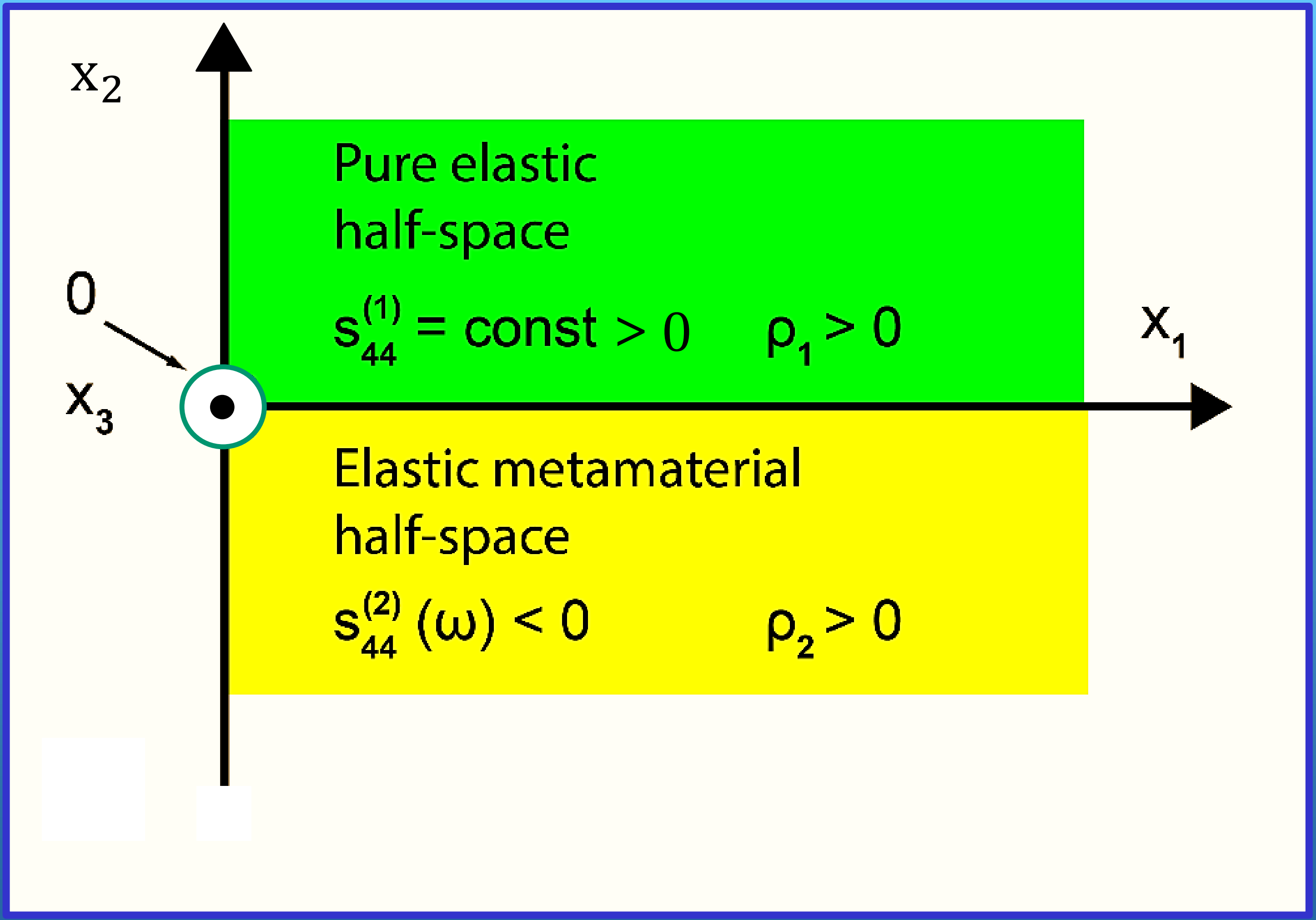


Fig. 3. Cross-section of the waveguide supporting the new proposed SH elastic surface waves propagating in the direction x_1, with exponentially decaying fields in the transverse direction x_2.

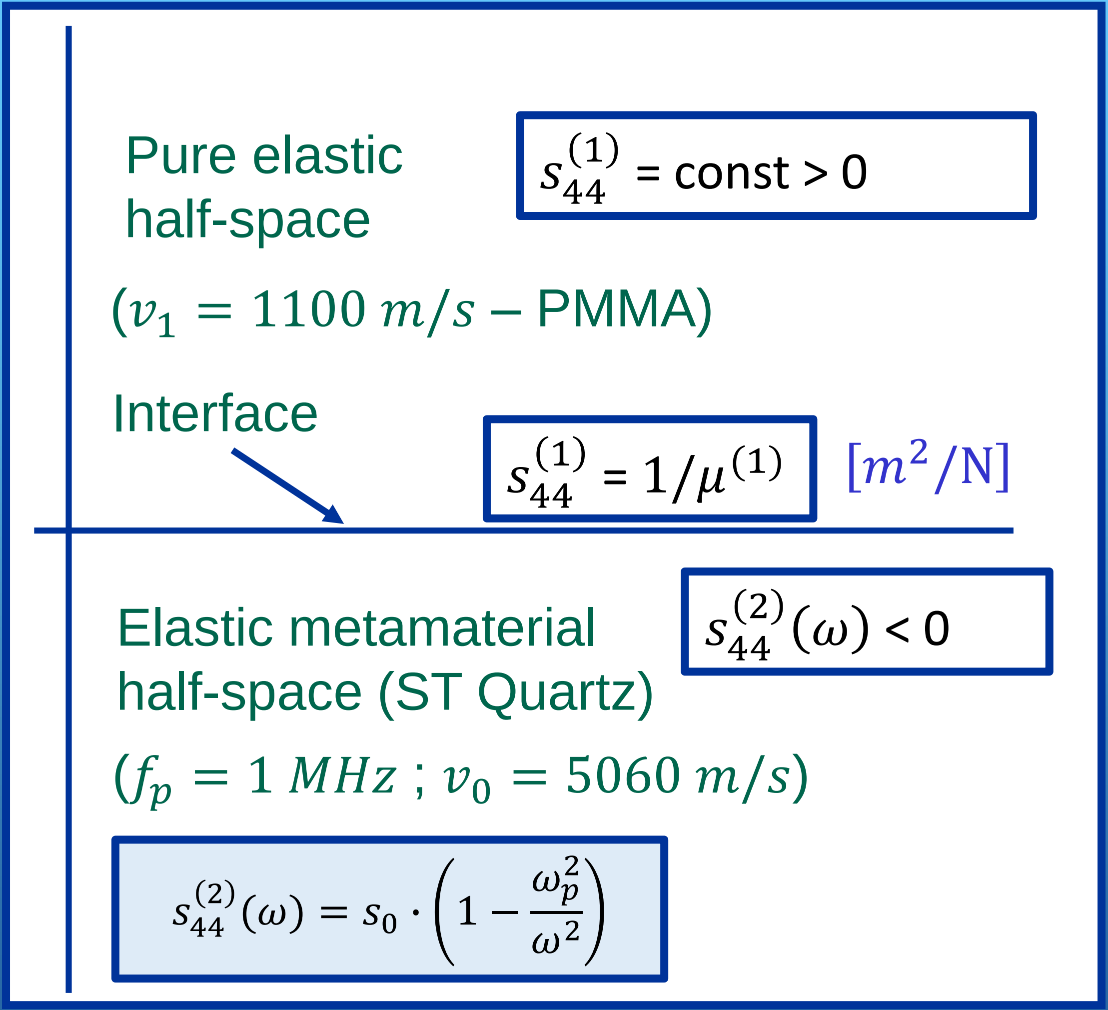
The mechanical displacement u_3 of the new SH surface elastic wave is polarized along the x_3 axis.

ω_p = Angular frequency of local oscillators embedded in the lower half-space

$$s_{44}^{(2)}(\omega) = s_0 \cdot \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

Drude's model

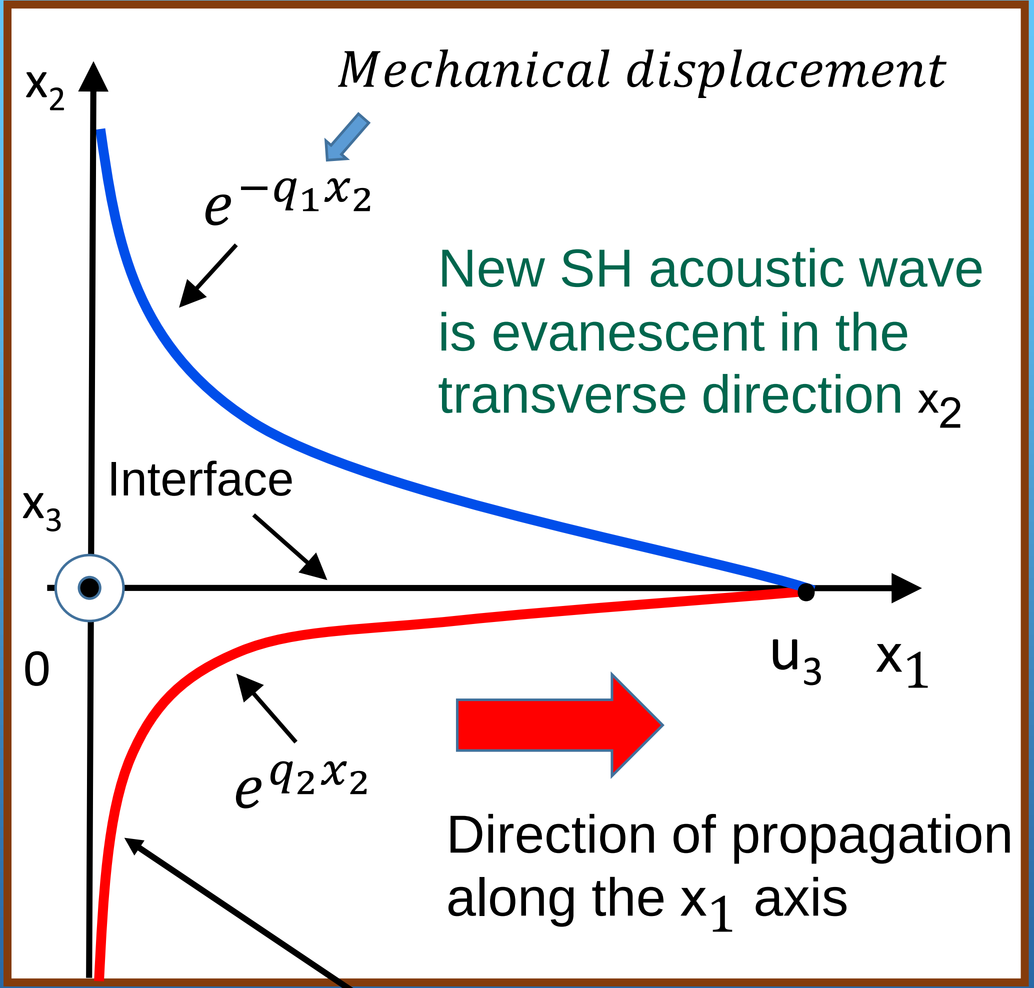
Newly discovered SH surface elastic wave propagating at the interface ($x_2 = 0$) between elastic semi-space and metamaterial elastic semi-space



2022

Fig.4.

Fig.5.



$s_{44}^{(2)}(\omega)$ follows Drude's model

Only one SH component of mechanical displacement u_3 along the x_3 axis.

$$s_{44}^{(2)}(\omega) = s_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$$

Fig.6.

Elastic compliance $s_{44}^{(2)}(\omega)$ of an elastic metamaterial substrate follows Drude's model.
Plasmonic elastic materials.
Meta – in Greek means: after, beyond
E.g., Metaphysics = After Physics

As can be seen, the elastic compliance $s_{44}^{(2)}(\omega)$ is negative in the range $0 < \omega < \omega_p$.

Movement of material particles is governed by the equation of motion along with the appropriate boundary conditions at the waveguide interface.

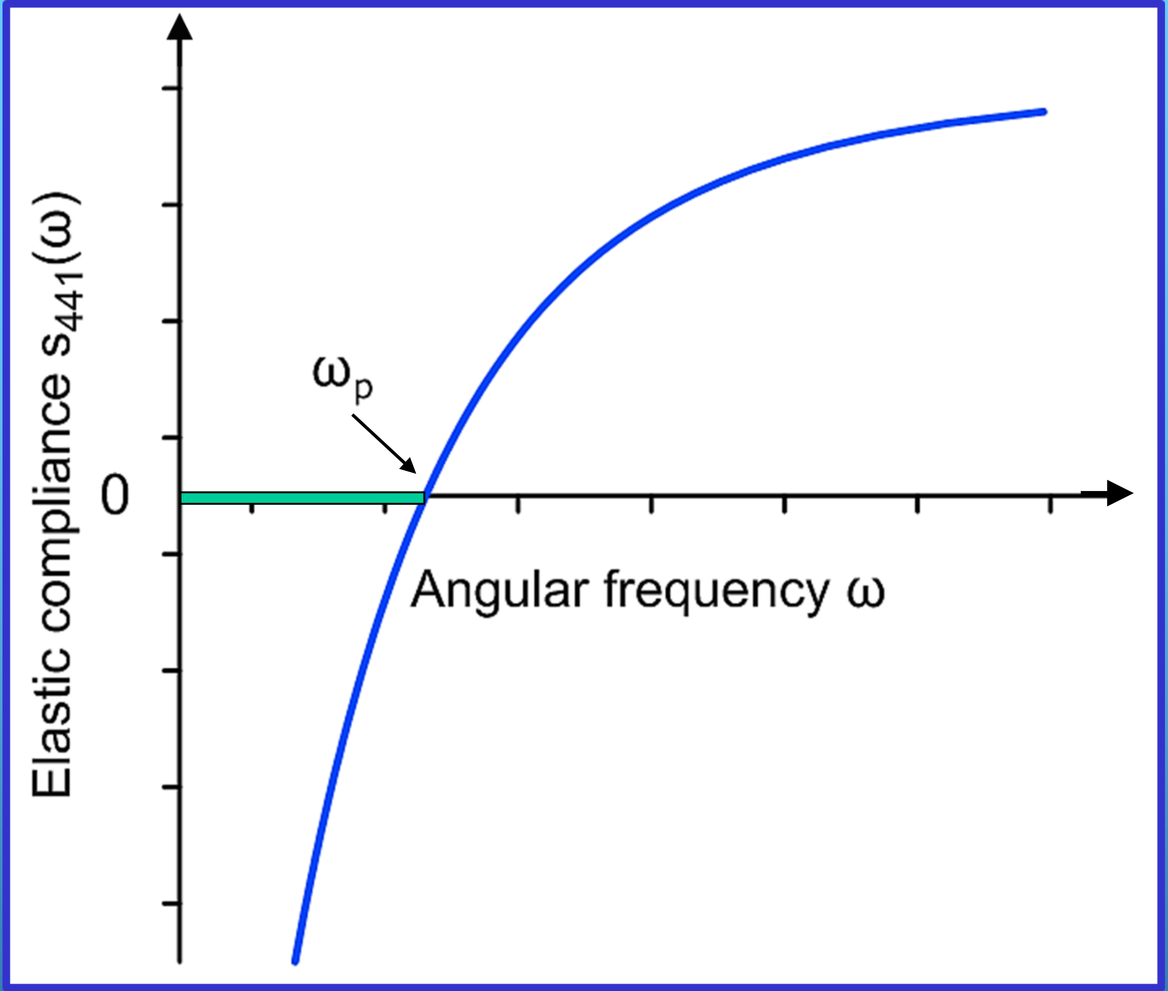


Fig.6. Elastic compliance $s_{44}^{(2)}(\omega)$ of an elastic metamaterial substrate (half-space) as a function of angular frequency ω.

MATHEMATICAL MODEL

1. Assumptions:

a) half-spaces are: linear, isotropic, lossless and homogeneous
b) there is no variations along the x_3 axis
c) absence of body forces: $F = \rho g$

We started from first principles: NEWTON'S second law of dynamics: μ = shear modulus

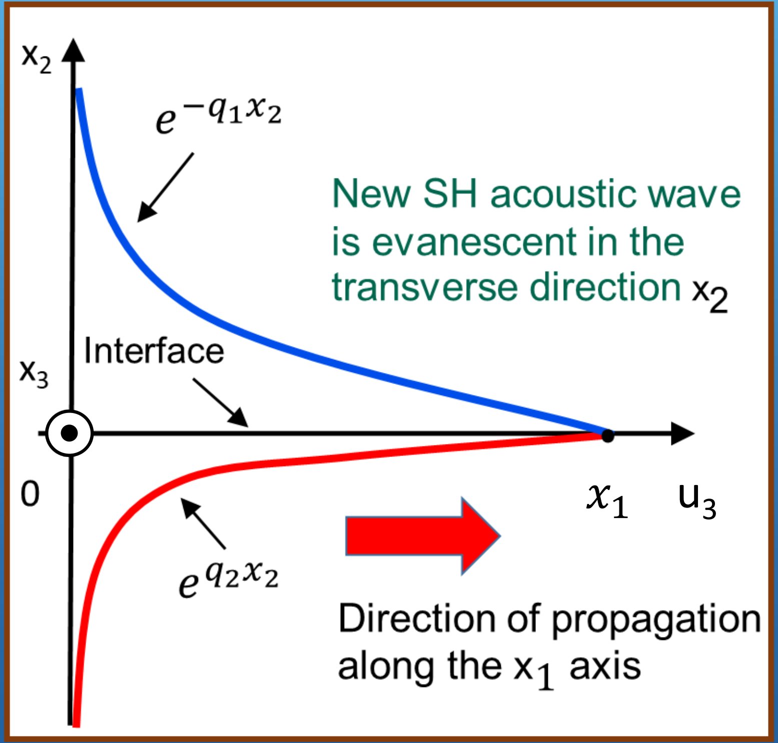


Fig.7.

$$1/\mu^{(1)} = s_{44}^{(1)} \quad : \quad 1/\mu^{(2)}(\omega) = s_{44}^{(2)}(\omega) \quad (1)$$

$$u_3^{(1)} = \mathrm{A} \cdot e^{-q_1 x_2}\, exp[j(k \cdot x_1 - \omega t)] \quad (2)$$

$$u_3^{(2)} = \mathrm{B} \cdot e^{q_2 x_2}\, exp[j(k \cdot x_1 - \omega t)] \quad (3)$$

Constitutive equations (relations):

$$\tau_{13} = 1/s_{44}\, \partial u_3/\partial x_1 \quad (4)$$

$$\tau_{23} = 1/s_{44}\, \partial u_3/\partial x_2 \quad (5)$$

- 2. Equations of motion (Wave equation):

(6)
$$\frac{1}{v_{1,2}^2}\frac{\partial^2 u_3^{(1,2)}}{\partial t^2} = \frac{\partial^2 u_3^{(1,2)}}{\partial x_1^2} + \frac{\partial^2 u_3^{(1,2)}}{\partial x_2^2}$$

where: u_3 is the mechanical displacement of the new SH elastic surface wave

- 3. Boundary conditions:

(7)
$$u_3^{(1)}\Big|_{x_2=0} = u_3^{(2)}\Big|_{x_2=0} \qquad \tau_{23}^{(1)}\Big|_{x_2=0} = \tau_{23}^{(2)}\Big|_{x_2=0}$$

- 4. Dispersion equation (Eigenvalue problem)

(8)
$$\frac{q_2}{-s_{44}^{(2)}(\omega)} = \frac{q_1}{s_{44}^{(1)}}$$

where: $s_{44}^{(1)} > 0$; $q_1 > 0$ *and* $q_2 > 0$ ⇨ $s_{44}^{(2)}(\omega) < 0$

$$q_1 = \sqrt{k^2 - \rho_1 \omega^2 s_{44}^{(1)}} \ \ and \ \ q_2 = \sqrt{k^2 - \rho_2 \omega^2 s_{44}^{(2)}}$$
$= transversal\ wavenumbers$

$$\Omega(k,\omega) = 0$$

Implicit function of ω and k

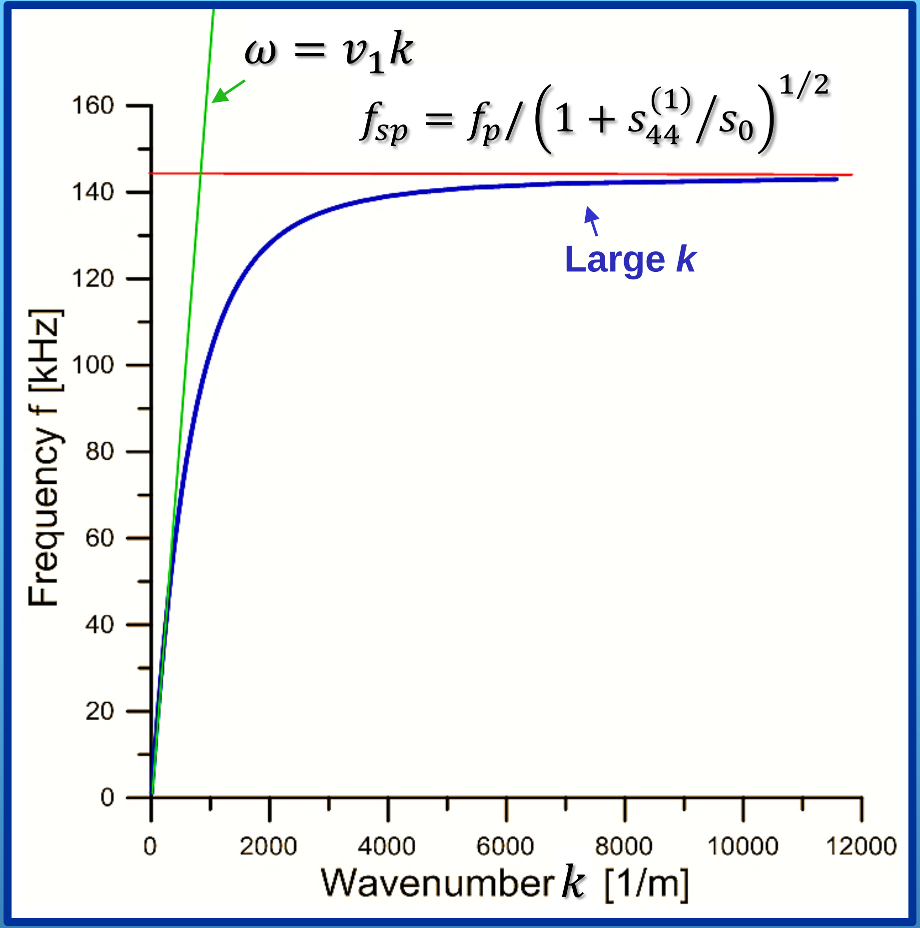


Fig.8.

Fig.8. Dispersion curve (angular frequency ω as a function of wavenumber k

ANALYTICAL SOLUTIONS:

(9)

$$f_{sp} = f_p/\left(1 + s_{44}^{(1)}/s_0\right)^{1/2}$$

Surface plasmon frequency

1) Dispersion equation

(10)

$$k(\omega) = \omega \cdot \sqrt{\frac{s_{44}^{(1)} \cdot s_{44}^{(2)}(\omega)}{\left(s_{44}^{(1)} + s_{44}^{(2)}(\omega)\right)}} \cdot \sqrt{\frac{s_{44}^{(1)} \cdot \rho_2 - s_{44}^{(2)}(\omega) \cdot \rho_1}{\left(s_{44}^{(1)} - s_{44}^{(2)}(\omega)\right)}}$$

2) Phase velocity

(11)

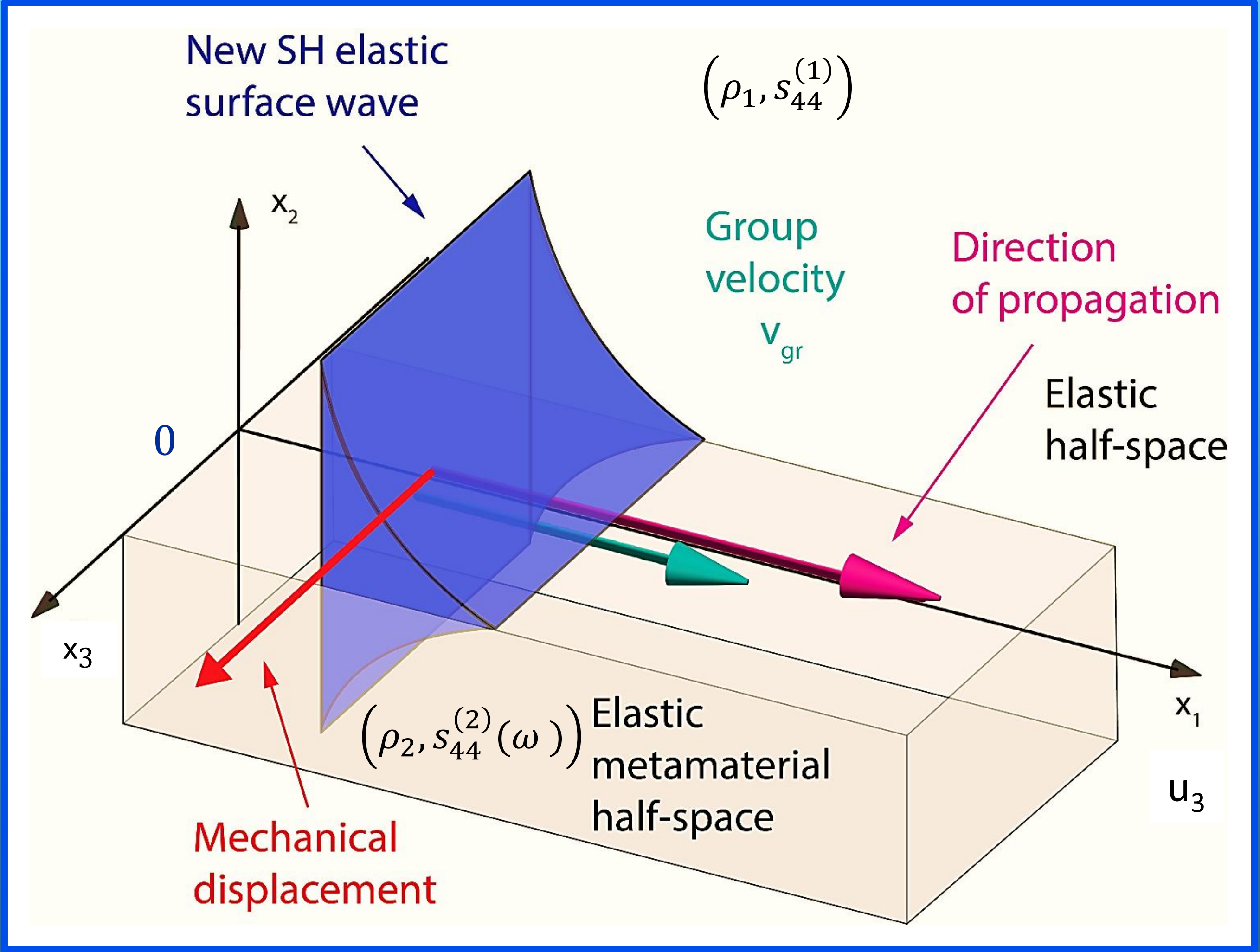
$$v_p(\omega) = \frac{\omega}{k} = \sqrt{\frac{\left(s_{44}^{(1)} + s_{44}^{(2)}(\omega)\right)}{s_{44}^{(1)} \cdot s_{44}^{(2)}(\omega)}} \cdot \sqrt{\frac{\left(s_{44}^{(1)} - s_{44}^{(2)}(\omega)\right)}{\left(s_{44}^{(1)} \cdot \rho_2 - s_{44}^{(2)}(\omega) \cdot \rho_1\right)}}$$

3) Group velocity: Differentation of an implicit function Ω

$$v_{gr}(\omega) = \left(\frac{d\omega}{dk}\right) = \frac{1}{v_p(\omega)} \frac{2 \cdot \left(\left(s_{44}^{(1)}\right)^2 - \left(s_{44}^{(2)}(\omega)\right)^2\right)}{2 s_{44}^{(1)} s_{44}^{(2)}(\omega) \left(s_{44}^{(1)} \cdot \rho_2 - s_{44}^{(2)}(\omega) \cdot \rho_1\right) + \omega s_{44}^{(1)} \frac{d s_{44}^{(2)}(\omega)}{d\omega} \left\{ \left(s_{44}^{(1)} \cdot \rho_2 - 2 s_{44}^{(2)}(\omega) \cdot \rho_1\right) + 2 \left(s_{44}^{(2)}(\omega)\right)^2 \frac{\left(s_{44}^{(1)} \cdot \rho_2 - s_{44}^{(2)}(\omega) \cdot \rho_1\right)}{\left(s_{44}^{(1)}\right)^2 - \left(s_{44}^{(2)}(\omega)\right)^2} \right\}}$$

(12)

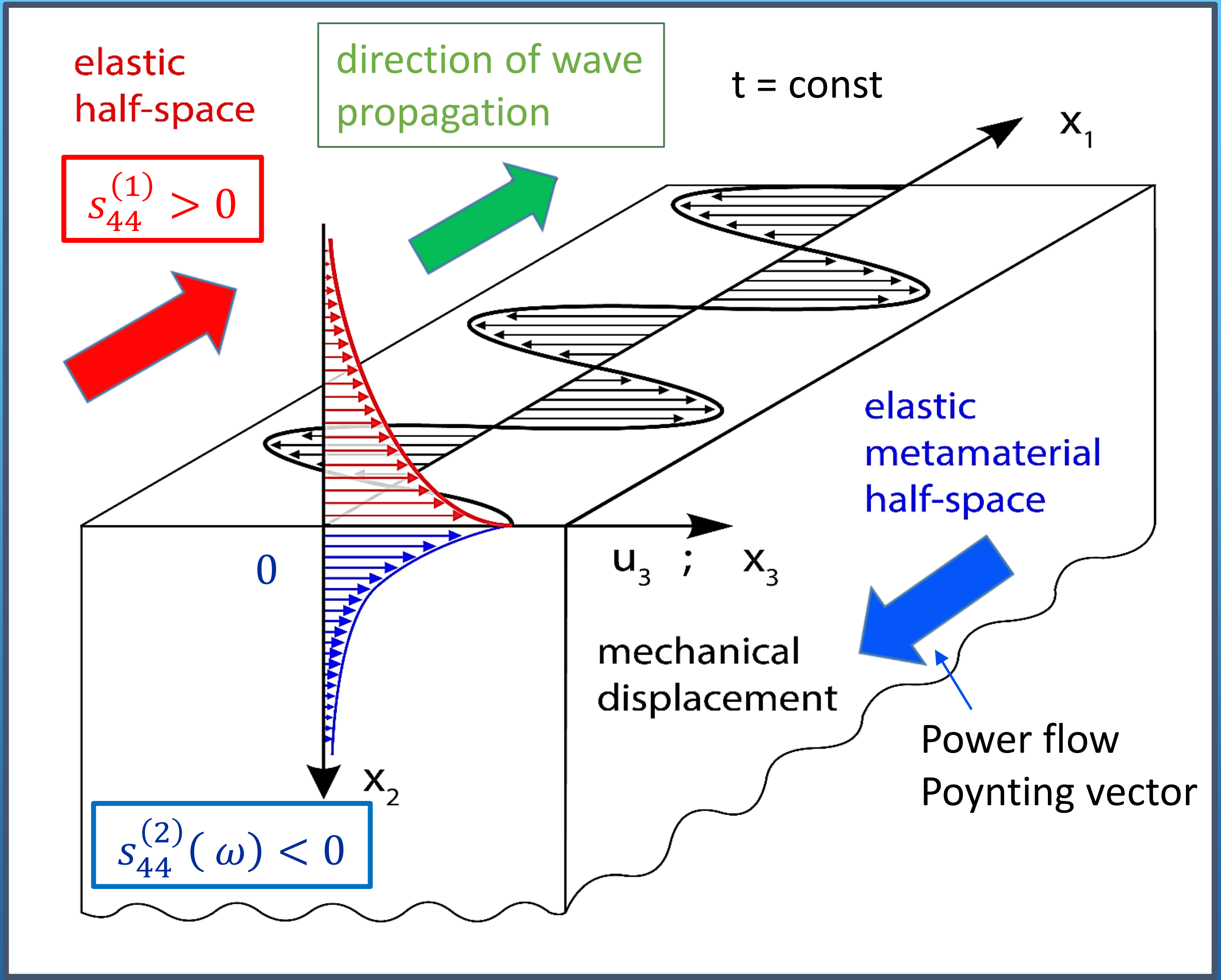
NEWLY DISCOVERED (SH) SURFACE ELASTIC WAVE MECHANICAL DISPLACEMENT DISTRIBUTION



Layered waveguide of the new Shear Horizontal (SH) surface elastic wave

Fig.9.

Mechanical displacement of the new wave

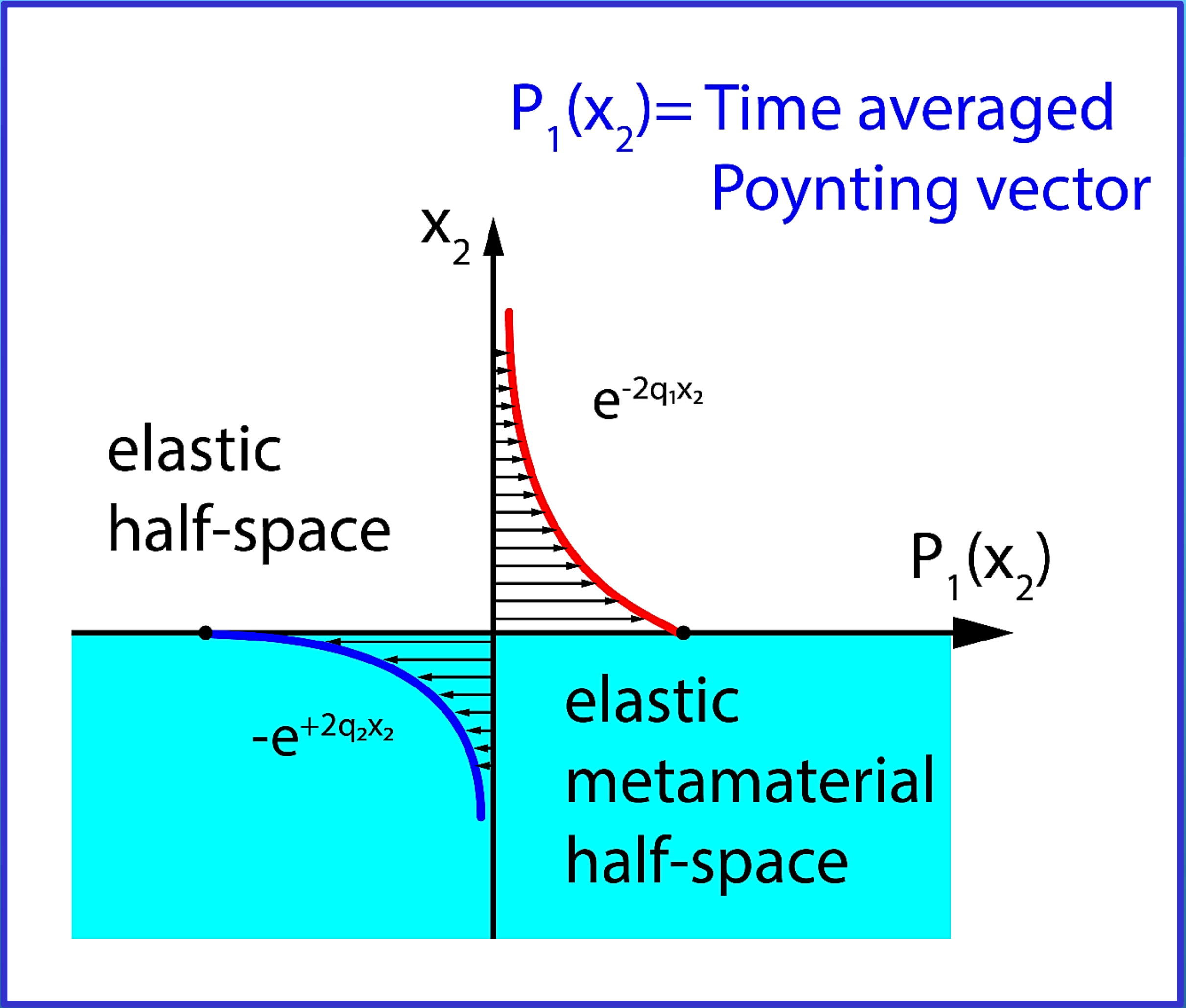


Lower metamaterial elastic half-space

Upper pure elastic half-space

Power flow in the new wave waveguide

Fig.10.



$$P_1 = -\frac{1}{2}[\tau_{13}(-j\omega u_3)^*]$$

Complex Poynting vector

$$P_1^{upper}(x_2) = \frac{1}{2}A^2\frac{1}{s_{44}^{(1)}}k(\omega)\cdot\omega\cdot\exp(-2q_1x_2)$$

$$P_1^{lower}(x_2) = \frac{1}{2}A^2\frac{1}{s_{44}^{(2)}(\omega)}k(\omega)\cdot\omega\cdot\exp(2q_2x_2)$$

Total Power Flow:

$$P_1^{total}(\omega) = \int_{-\infty}^{0} P_1^{lower}(x_2)\,dx_2 + \int_0^{+\infty} P_1^{upper}(x_2)\,dx_2 = \frac{1}{4}A^2k(\omega)\cdot\omega\cdot\left(\frac{1}{s_{44}^{(2)}(\omega)}\frac{1}{q_2}+\frac{1}{s_{44}^{(1)}}\frac{1}{q_1}\right)$$

$$P_1^{total} \sim v_e = v_{gr} \underset{\omega\to\omega_{sp}}{\rightarrow} 0\,;\ v_e = energy\ velocity$$

Intriguing Properties

1) Phase velocity $v_p(\omega) \to 0$ for $\omega \to \omega_{sp}$

2) Group velocity $v_{gr}(\omega) \to 0$ for $\omega \to \omega_{sp}$

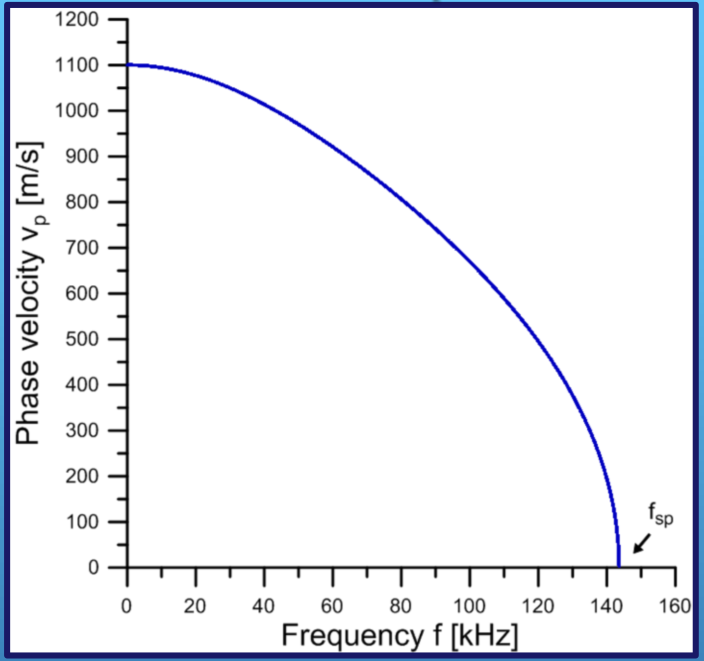
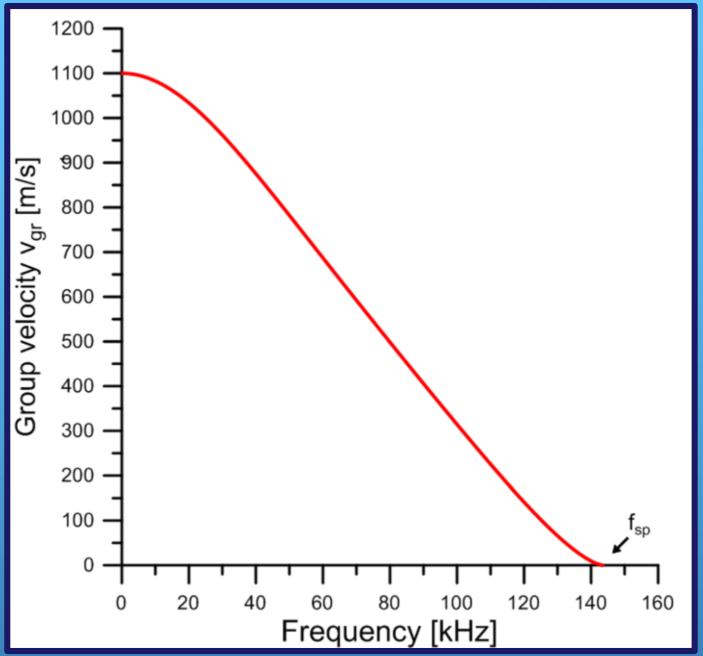


Fig.11.



Remark:
Newly discovered SH surface elastic wave is a direct analogue of the Surface Plasmon Polariton (SPP) TM electromagnetic wave

Fig.12.

f [kHz]	v_p [m/s]	v_{gr} [m/s]	δ_{upper} [mm]	δ_{lower} [mm]	λ_{pl} [mm]	λ_0[mm]
10	1094.2	1082.7	169.3	0.805	109.4	110.0
20	1077.1	1034.0	42.2	0.802	53.7	55.0
50	970.2	782.2	6.55	0.780	19.4	22.0
140	194.2	9.45	0.224	0.213	1.39	7.86
143	77.6	0.61	0.087	0.086	0.54	7.69

Table.2.

Extraordinary Features of the New Wave

1. Newly discovered SH surface elastic wave No. 1 is an elastic anologue of the Surface Plasmon Polariton (SPP) electromagnetic TM wave propagating at the interface: dielectric over metallic substrate.
   Here, MECHANICS meets ELECTROMAGNETISM

2. Ability to amplify evanescent waves

3. Near-field subwavelength acoustic imaging (resolution below the diffraction limit)
   In the table below, we can observe a very useful advantage of the new elastic wave

Table.3.

Type of wave	Resolution	Frequency
Using the newly discovered wave	of the order of μm	of the order of MHz
Using conventional elastic waves	of the order of μm	of the order of GHz

Resolution of the order of μm

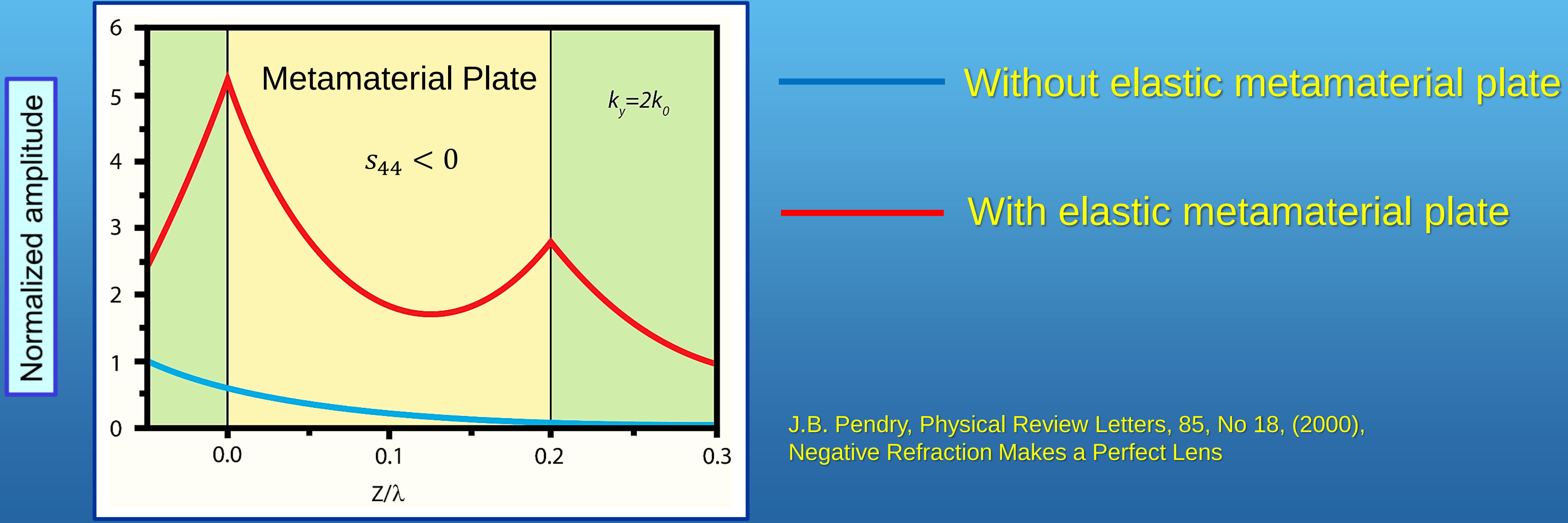
MHz not GHz

4. Breaking the diffraction limit

5. Wave trapping (zero group and energy velocity)

Amplification of evanescent waves (schematic view)

New SH elastic wave inherits majority of extraordinary properties of Surface Plasmon Polariton wave (SPP), e.g., amplification of evanescent waves

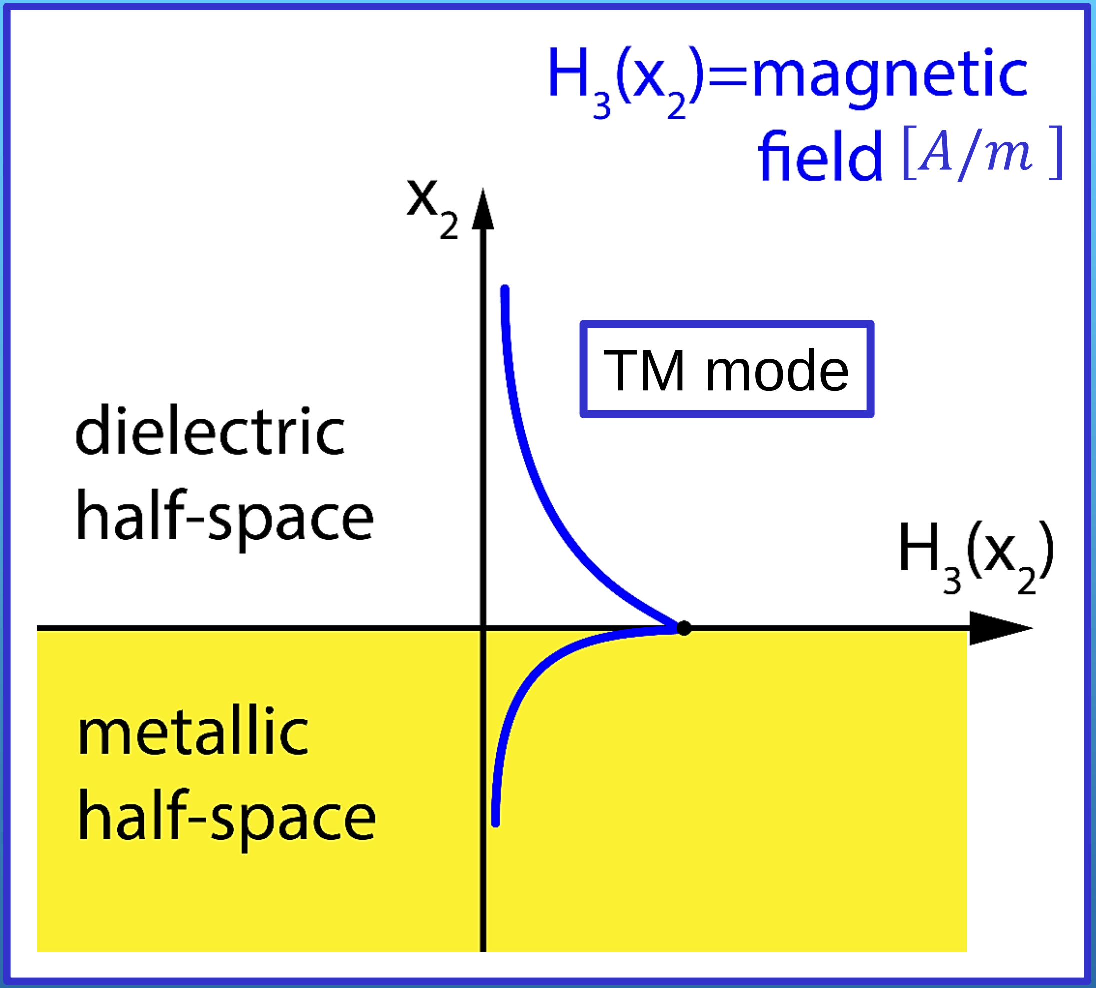


J.B. Pendry, Physical Review Letters, 85, No 18, (2000), Negative Refraction Makes a Perfect Lens

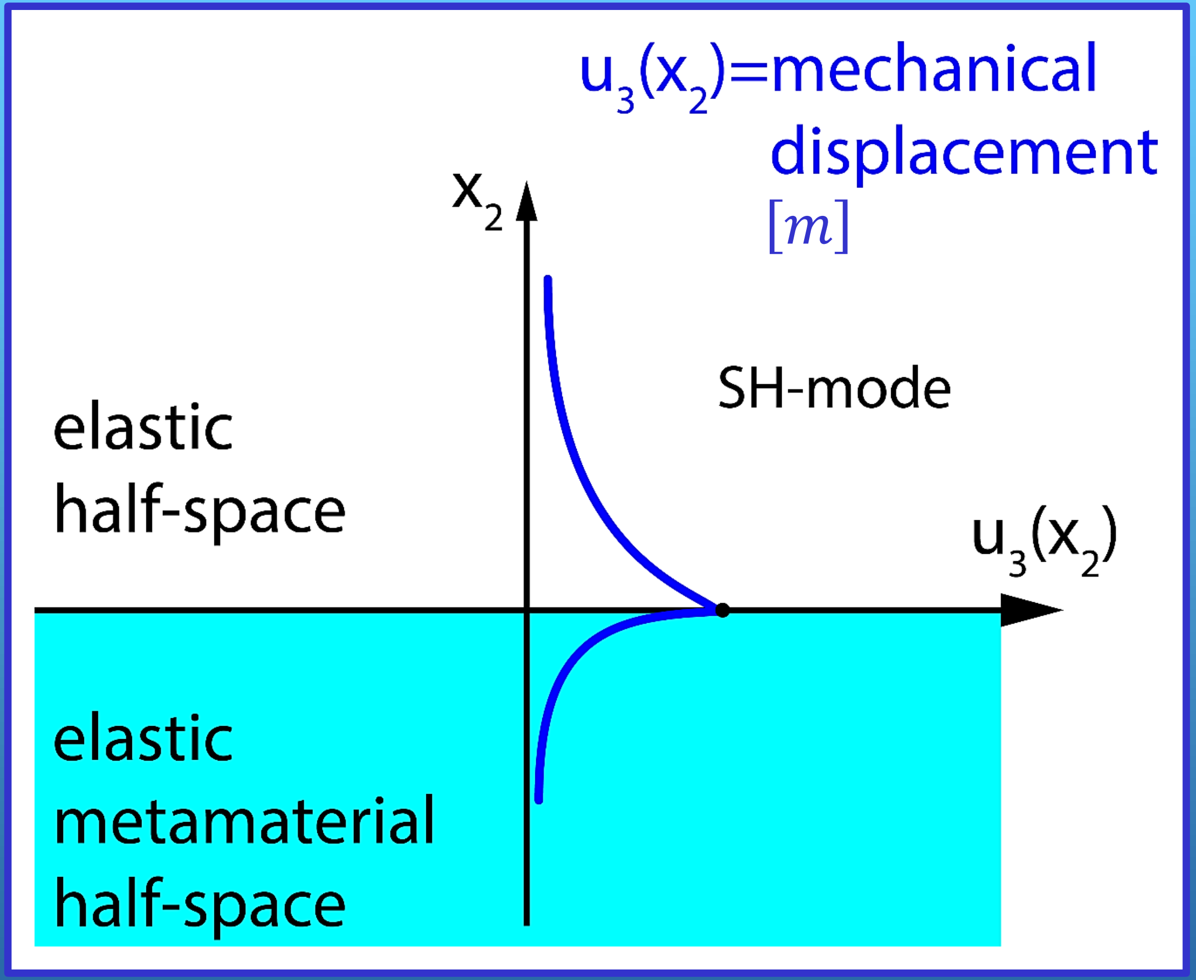
Fig.13.

Newly discovered SH surface elastic wave has an exact analogue in electromagnetism - Surface Plasmon Polariton (SPP) wave

Fig.14.



TM Electromagnetic Surface Wave



SH Elastic Surface Wave

Fig.15.

Direct Sturm-Liouville Problem
2 different physical phenomena share a Common Mathematical Model

Correspondence between:

1. Newly discovered SH Elastic Surface Waves and
2. Electromagnetic Surface Waves of the Surface Plasmon Polariton (SPP) type

Analogies between Mechanics and Electromagnetism
Here: Newly discovered SH surface elastic waves bridge the gap between MECHANICS and ELECTROMAGNETISM

These two waves have the same:

1. Field distributions
2. Geometry of the waveguide
3. Equations of motion
4. Boundary conditions
5. Dispersion equation
6. Analogous analytical solutions for field variables

TM modes (SPP): ELECTROMAGNETISM

Helmholtz equation resulting from Maxwell's equations

1. Dielectric half-space modes

$$\frac{d^2H_3}{dx_2{}^2} + \mu_1\varepsilon_1\omega^2 \cdot H_3 = k^2 \cdot H_3 \qquad (13)$$

2. Metallic half-space – TM modes

$$\frac{d^2H_3}{dx_2{}^2} + \mu_2\varepsilon_2(\omega)\omega^2 \cdot H_3 = k^2 \cdot H_3 \qquad (14)$$

3. Boundary conditions

a) continuity of $\frac{1}{\varepsilon}\frac{dH_3}{dx_2}$ and H_3 at $x_2 = 0$ (15)

4. Dispersion equation (Eigenvalue equation):

$$\frac{q_2}{-\varepsilon_2(\omega)} = \frac{q_1}{\varepsilon_1}$$

where: $q_1 = \sqrt{k^2 - \mu_1\varepsilon_1\omega^2}$

$q_2 = \sqrt{k^2 - \mu_2\varepsilon_2(\omega)\omega^2}$ (16)

New SH surface elastic wave: MECHANICS

Helmholtz equation resulting from Equation of motion

1. Elastic half-space

$$\frac{d^2u_3}{dx_2{}^2} + \rho_1 s_{44}^{(1)}\omega^2 \cdot u_3 = k^2 \cdot u_3 \qquad (17)$$

2. Elastic metamaterial half-space

$$\frac{d^2u_3}{dx_2{}^2} + \rho_2 s_{44}^{(2)}(\omega)\omega^2 \cdot u_3 = k^2 \cdot u_3 \qquad (18)$$

3. Boundary conditions

a) continuity of $\frac{1}{s_{44}}\frac{du_3}{dx_2}$ and u_3 at $x_2 = 0$ (19)

4. Dispersion equation (Eigenvalue equation):

$$\frac{q_2}{-s_{44}^{(2)}(\omega)} = \frac{q_1}{s_{44}^{(1)}}$$

where: $q_1 = \sqrt{k^2 - \rho_1 s_{44}^{(1)}\omega^2}$

$q_2 = \sqrt{k^2 - \rho_2 s_{44}^{(2)}(\omega)\omega^2}$ (20)

Table 4. Correspondence between field variables of the SPP electromagnetic waves propagating in metal–dielectric waveguides and the proposed new SH elastic surface waves propagating in elastic metamaterial waveguides.

No	SPP electromagnetic surface waves in metal-dielectric waveguides		New SH elastic surface waves in elastic metamaterial waveguides	
	Property	Implementation	Implementation	Property
1	Longitudinal electric field	E_1	τ_{23}	Shear horizontal SH mechanical stress
2	Transverse electric field	E_2	$-\tau_{13}$	Shear mechanical stress
3	transverse magnetic field	H_3	$v_3 = -j\omega u_3$	SH particle velocity $v_3 = \partial u_3/\partial t$
4	Dielectric function in metal	$\varepsilon_2(\omega)$	$s_{44}^{(2)}(\omega)$	Elastic compliance in metamaterial half-space
5	Dielectric function in dielectric	ε_1	$s_{44}^{(1)}$	Elastic compliance in conventional half-space
6	Magnetic permeability in metal	μ_1	ρ_1	Density of metamaterial half-space
7	Magnetic permeability in dielectric	μ_2	ρ_2	Density of conventional half-space
8	Wavenumber for $\mu_1/\mu_2 = 1$	$k(\omega) = k_1\sqrt{\frac{\varepsilon_2(\omega)}{\varepsilon_2(\omega)+\varepsilon_1}}$	$k(\omega) = k_1\sqrt{\frac{s_{44}^{(2)}(\omega)}{s_{44}^{(2)}(\omega)+s_{44}^{(1)}}}$	Wavenumber for $\rho_1/\rho_2 = 1$
9	Phase velocity of SPP electromagnetic waves	$v_p(\omega) = v_1\sqrt{\frac{\varepsilon_2(\omega)+\varepsilon_1}{\varepsilon_2(\omega)}}$	$v_p(\omega) = v_1\sqrt{\frac{s_{44}^{(2)}(\omega)+s_{44}^{(1)}}{s_{44}^{(2)}(\omega)}}$	Phase velocity of new SH elastic surface waves
10	Complex Poynting vector in propagation direction x_1	$P_1 = \frac{1}{2}E_2 \times H_3^*$	$P_1 = -\frac{1}{2}\tau_{13}v_3^*$	Complex Poynting vector in propagation direction x_1
11	Complex Poynting vector in transverse direction x_2	$P_2 = -\frac{1}{2}E_1 \times H_3^*$	$P_2 = -\frac{1}{2}\tau_{23}v_3^*$	Complex Poynting vector in transverse direction x_2

Analogies between Elastodynamics and Electrodynamics

1) Various classical waves are governed via similar types of wave equation

2) Analogies between those waves are very fruitful and repeatedly resulted in the mutual export of ideas between optics and acoustics

3) Surface waves at interfaces between continuos media, such as Surface Plasmon Polariton (SPP) are highly important for modern optics

4) Surface modes are particularly important in the context of topological quantum or classical-wave systems, which are currently attracting enormous attention

5) The equations for SH elastic waves in solid media also have a form compatible to Maxwell equations (ε, μ) and also involve two medium parameters: the density and mechanical compliance (ρ, s_{44})

6) Many acoustic and electromagnetic quantities (e.g., the Energy density, Poynting vector etc.) have similar forms

SUMMARY

1) New SH surface elastic wave was recently discovered. It has unique properties:

   a) deeply subwavelength $\sim\lambda/20$ penetration depth (breaking the diffraction limit)
   b) high concentration of energy in the vicinity of the guiding surface
   c) very low phase and group velocities tending to zero $(v_p \text{ and } v_{gr} \to 0\,;\,\omega \to \omega_{sp})$
   d) can be used in near-field subwavelenght acoustic imaging and superlensing

2) Strict analogies between Mechanical waves and Electromagnetic waves were found

3) A new branch of science – Plasmonic Elastodynamics has been developed.

4) Interdisciplinary connections between Mechanics (Elastodynamics), Electrodynamics and Quantum Mechanics were identified

5) Nowadays, we observe a tremendous new development in Acoustic (Mechanical) waves

6) From Electrodynamics ⇨ Mechanics = Metamaterials and Photonic (Phononic) Crystals

7) From Quantum Mechanics ⇨ Mechanics = Topological Materials and Non-Hermitian Systems

THE FOLLOWING METAMATERIALS WERE FABRICATED, UP TO DATE

1) Spring compliance: $C(\omega) = C_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$

2) Mass density: $\rho(\omega) = \rho_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$

3) Bulk modulus: $B(\omega) = B_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$

4) Negative: elastic compliance: $s_{44}(\omega) < 0$; papers:

a) Y. Lai et.al., Nature Materials, 10, 620, 2011;
b) X. Zhou, X. Liu and G. Hu, Theoretical & Applied Mechanical Letters, 2, 041001, 2012

5) Negative Young's modulus $\mathrm{E}(\omega) < 0$: International Journal of Engineering Science 150 (2020) 103231
S. Adhikari et al.,

6) Drude-like elastic compliance:

$s_{44}(\omega) = s_0 \cdot \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$ (?)

Spring-mass model of a mechanical resonator

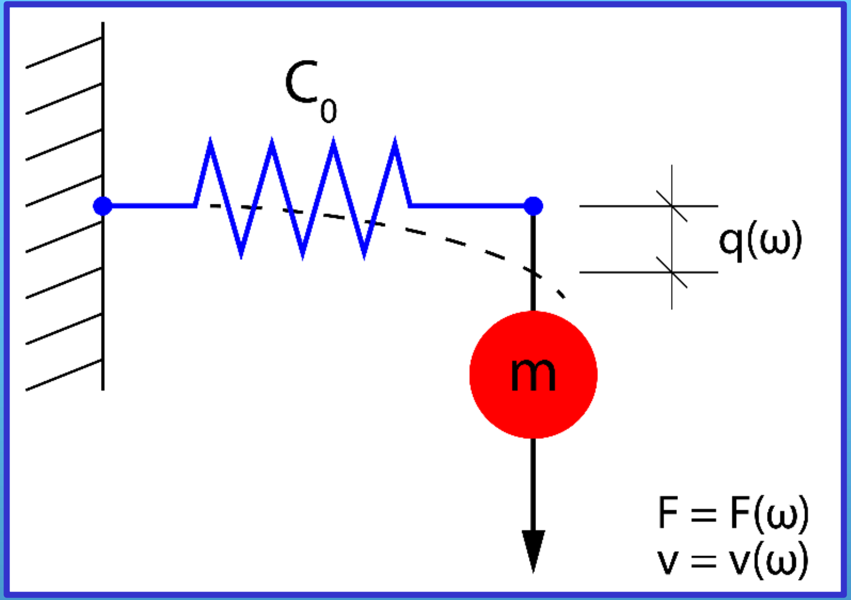


Fig.16.

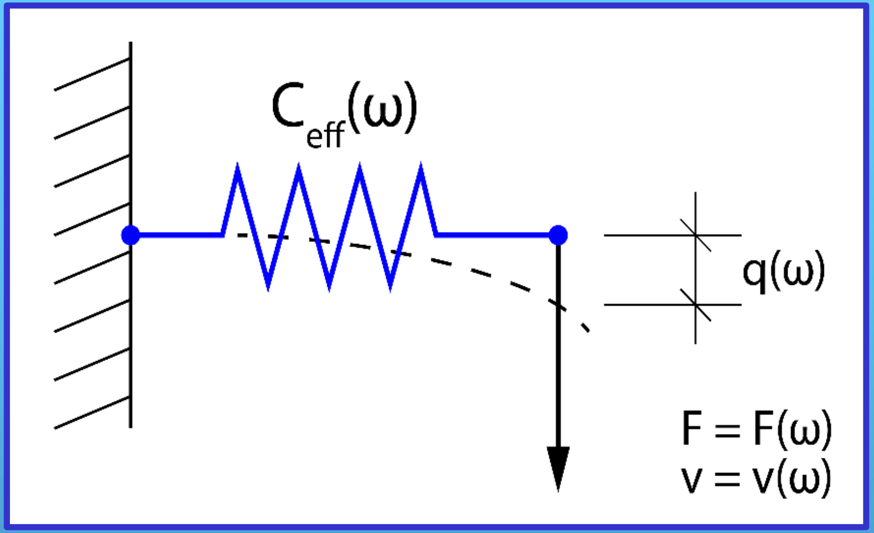


Fig.17.

Fig.16. Spring-mass model of a mechanical resonator $C_0 = 1/k$ spring elastic compliance.

$$Y(\omega) = v(\omega)/F(\omega) = j\omega C_0 + \frac{1}{j\omega m} = j\omega C_0\left(1 - \frac{\omega_0^2}{\omega^2}\right)$$

$Y(\omega)$ = Mechanical admittance

Fig.17. Equivalent lumped elastic compliance $C_{eff}(\omega)$ representing an overall behavior of the mechanical resonator from Figure 16.

$$C_{eff}(\omega) = C_0\left(1 - \frac{\omega_0^2}{\omega^2}\right)$$

$C_{eff}(\omega)$ = Drude's model

Mechanical Metamaterials

1. With negative mass density: $\rho < 0$
2. With negative stiffness: negative slope in their stress-strain relations
3. With negative compressibility: B < 0
4. Dilatational metamaterials: auxetic metamaterials, Poisson's ratio = -1
5. Pentamode metamaterials: G = 0, $\nu = 0.5$ – metafluids
6. Auxetic metamaterials: $\nu < 0$ – negative Poisson's ratio

E. Barchiesi et al.,
Mechanical metamaterials: a state of the art
•February 2018
•Mathematics and Mechanics of Solids
24(10):108128651773569
DOI:10.1177/1081286517735695