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Effect of a viscous liquid loading on Love wave propagation

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ABSTRACT

This paper describes a theory of surface Love waves propagating in a layered elastic waveguide loaded on its surface by a viscous (Newtonian) liquid. An analytical expression for the complex dispersion equation of Love waves has been established. The real and imaginary parts of the complex dispersion equation were separated and resulting system of nonlinear algebraic equations was solved numerically. The influence of the viscosity of liquid on the dispersion curves of phase velocity, the wave attenuation and the distribution of the Love wave amplitude is analyzed numerically. The propagation loss is produced only by the viscosity of liquids. Elastic layered waveguide is assumed to be loss-less. The numerical solutions show the dependence of the phase velocity change, the wave attenuation and the wave amplitude distribution in terms of the liquid viscosity and the wave frequency. The results of the investigations are fundamental and can be applied in the design and development of liquid viscosity sensors and biosensors, in Non-Destructive Testing (NDT) of materials, in geophysics and seismology.

1. Introduction

Love waves have been applied successfully in many of the scientific and technological fields, such as geophysics, seismology and earthquake engineering (Luo et al., 2010; Bautista and Stoll, 1995; Boxberger et al., 2011; Fukao and Abe, 1971), nondestructive testing and material characterization (Kuznetsov, 2010; Kiełczyński and Szalewski, 2011). In recent years, Love-wave-based devices have been developed for use as viscosity sensors (Kiełczyński and Płowiec, 1989; Rostocki et al., 2010; Kiełczyński et al., 2011; Raimbault et al., 2008; Chen and Liu, 2010), biosensors (Länge et al., 2008; Oh et al., 2009), and chemical sensors (Wang et al., 2008).

There are a number of papers concerning the propagation of acoustic surface waves in waveguides covered with a viscous liquid (e.g. Kim, 1992; Wu and Wu, 2000; Guo and Sun, 2008). However a detailed quantitative analysis of Love wave propagation in waveguides loaded with Newtonian liquid is still lacking.

Love wave is a transverse surface wave having one component of mechanical displacement, which is parallel to the surface and perpendicular to the direction of wave propagation. Love waves propagate in an elastic layered structures (waveguides) consisting of an elastic surface layer rigidly bonded to the elastic substrate. The condition for the existence of Love waves is that the bulk transverse wave velocity in the layer is lower than that in the substrate. It was Love who first put forward the theory of this type of waves in 1911.

In this study, the surface of the waveguide is in contact with a viscous liquid. Love waves propagating in such waveguides undergo attenuation, hence, the wave number of the Love wave becomes complex:

$$k = k_0 + j\alpha \tag{1}$$

where $j = \sqrt{-1}$.

The real part of the wave number k_0 determines the phase velocity of the Love wave. The imaginary part of the wave number α is an attenuation coefficient of the Love wave.

In this paper we perform a rigorous mathematical analysis of the problem of the Love wave propagation in the waveguides covered with a viscous liquid. The effect of the liquid viscosity η on the phase velocity of Love waves and attenuation is presented and analyzed. Subsequently, the plots of phase velocity and attenuation of Love waves in function of frequency for various values of viscosity are given. Moreover, the dependencies of the Love wave amplitude versus the distance from the waveguide surface are established and plotted.

The results obtained in this paper are fundamental and can provide essential data for the design and development of Love-wave-based liquid sensing devices.

2. Mathematical formulation of the problem

2.1. Physical model

The waveguide structure in which the Love wave propagates consists of a loss-less elastic surface layer deposited on a loss-less elastic substrate, see Fig. 1.

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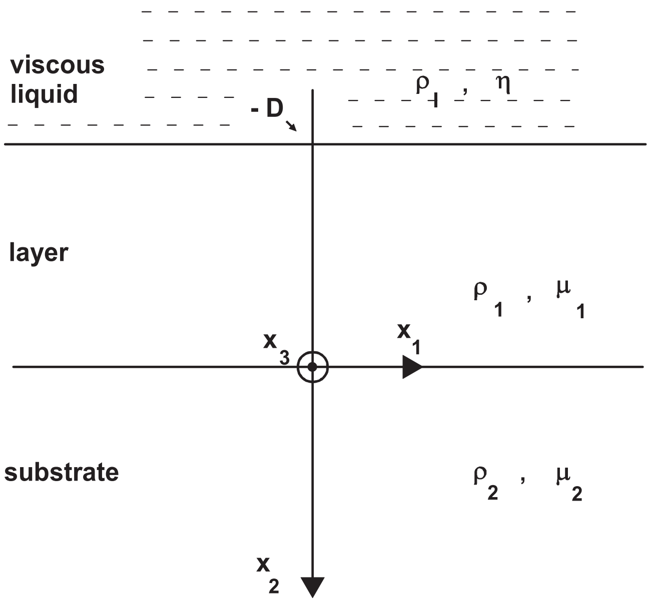

Fig. 1. Geometry of a Love wave waveguide. The waveguide surface ($x_2 = -D$) is in contact with a semi-infinite viscous liquid. Here, μ_1, μ_2 and ρ_1, ρ_2 correspond respectively, to the shear modulus and mass density in the surface layer (index 1) and in the substrate (index 2). η and ρ_l are the viscosity and mass density of a viscous liquid. x_1 is the direction of propagation. Love wave is polarized along the x_3 direction. D is the thickness of the elastic surface layer.

Bulk transverse wave velocity in the layer $v_1 = (\mu_1/\varrho_1)^{1/2}$ is smaller than the corresponding shear wave velocity $v_2 = (\mu_2/\varrho_2)^{1/2}$ in the substrate. The waveguide surface is loaded with a viscous (Newtonian) liquid that occupies half-space ($x_2 < -D$). The mechanical displacement of the Love wave u_3 is polarized along the x_3 axis, perpendicular to the direction of propagation x_1. The waveguide surface is at $x_2 = -D$. The considered problem is two-dimensional, having no variation along the x_3 axis.

The Love wave exhibits a multimode character. In many practical applications (e.g., in sensors and NDT) the most important is the fundamental mode of Love waves. Therefore, in this study we have restricted our attention to the propagation of the fundamental (the lowest) mode of Love waves.

Love waves can be generated experimentally using, for example a piezoelectric plate transducer (Kiełczyński and Szalewski, 2011). Plate transducer (e.g., made of piezoelectric PZT ceramics) is rigidly bonded to the Love wave waveguide face and excited to shear vibrations parallel to the surface of the waveguide (along the x_3 axis). Generated in this way impulses of the Love wave propagate along the waveguide surface (along the x_1 axis). Theoretical and experimental analysis of the generation of SH (shear horizontal) surface waves with a plate transducer is shown in (Kinh and Pajewski, 1980).

2.2. Governing differential equations

2.2.1. Viscous liquid region ($x_2 < -D$)

The velocity field v_3 (of the SH acoustic wave) in a viscous liquid ($x_2 < -D$) is governed by the Navier–Stokes equation:

$$\frac{\partial v_3}{\partial t} - \frac{\eta}{\rho_l}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) v_3 = 0 \qquad (2)$$

where η is the viscosity and ρ_l density of a liquid.

2.2.2. Elastic surface layer ($0 > x_2 > -D$)

The mechanical displacement field u_3 (of the SH acoustic wave) in the surface layer ($0 > x_2 > -D$) fulfills the following equation of motion:

$$\frac{1}{v_1^2}\frac{\partial u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) u_3 \qquad (3)$$

where $v_1 = (\mu_1/\rho_1)^{1/2}$ is the bulk shear wave velocity in the layer.

2.2.3. Elastic substrate ($x_2 > 0$)

The mechanical displacement field u_3 (of the SH acoustic wave) in the substrate ($x_2 > 0$) satisfies the equation of motion as follows:

$$\frac{1}{v_2^2}\frac{\partial u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) u_3 \qquad (4)$$

where $v_2 = (\mu_2/\rho_2)^{1/2}$ is the bulk shear wave velocity in the substrate.

2.3. Propagation wave solution

2.3.1. Elastic surface layer ($0 > x_2 > -D$)

We postulate the following solution of the Eq. (3) describing the mechanical displacement field $u_3^{(1)}$ of the Love wave in the surface layer:

$$u_3^{(1)} = W(x_2) \cdot exp[j(k \cdot x_1 - \omega t)] \qquad (5)$$

where k is the complex wave number, ω is the angular frequency.

Substitution of Eq. (5) into Eq. (3) results in:

$$W''(x_2) - \left(k_1^2 - k_0^2\right) \cdot W(x_2) = 0 \qquad (6)$$

where the superscript prime denotes the differentiation with respect to x_2. The solution of Eq. (6) can be expressed as:

$$W(x_2) = C_1 \cdot \sin(q \cdot x_2) + C_2 \cdot \cos(q \cdot x_2) \qquad (7)$$

where

$$q = \left(k_1^2 - k^2\right)^{1/2}; \quad k_1 = \frac{\omega}{v_1}$$

C_1 and C_2 are arbitrary constants

The shear stress component that will be used in boundary conditions is given by:

$$\tau_{23}^{(1)} = \mu_1 \frac{\partial u_3^{(1)}}{\partial x_2} = C_1 \cdot \mu_1 \cdot q \cdot \cos(q \cdot x_2) - C_2 \cdot \mu_{1.} q \cdot \sin(q \cdot x_2) \qquad (8)$$

2.3.2. Elastic substrate ($x_2 > 0$)

We consider the following solution of Eq. (4) of the mechanical displacement field $u_3^{(2)}$ of the Love wave in the substrate:

$$u_3^{(2)} = U(x_2) \cdot exp[j(k \cdot x_1 - \omega t)] \qquad (9)$$

Substituting Eq. (9) into Eq. (4) yields:

$$U''(x_2) - \left(k^2 - k_2^2\right) \cdot U(x_2) = 0 \qquad (10)$$

As a solution of Eq. (10) we choose:

$$U(x_2) = C_3 \cdot exp(-b \cdot x_2) \qquad (11)$$

where

$$b = \left(k^2 - k_2^2\right)^{1/2}$$

$$k_2 = \frac{\omega}{v_2} \quad \text{and} \quad Re(b) > 0$$

C_3 is an arbitrary constant.

This solution represents SH surface wave that amplitude decays to zero, when $x_2 \to \infty$.

The shear stress component needed in the boundary condition is given by:

$$\tau_{23}^{(2)} = \mu_2 \frac{\partial u_3^{(2)}}{\partial x_2} = C_3 \mu_2 (-b) \cdot exp(-b \cdot x_2) \cdot exp[j(kx_1 - \omega t)] \qquad (12)$$

2.3.3. Viscous liquid region ($x_2 < -D$)

The solution of Eq. (2) of the velocity field v_3 (of the Love wave) in the viscous liquid is sought in the form:

$$v_3 = V(x_2) \cdot exp[j(k \cdot x_1 - \omega t)] \qquad (13)$$

Substitution of Eq. (13) into Eq. (2) gives:

$$V''(x_2) - \left(k^2 - j\omega \frac{\rho_l}{\eta}\right) \cdot V(x_2) = 0 \qquad (14)$$

Solving Eq. (14), we get:

$$V(x_2) = C_4 \cdot exp(\lambda_1 \cdot x_2) \qquad (15)$$

where

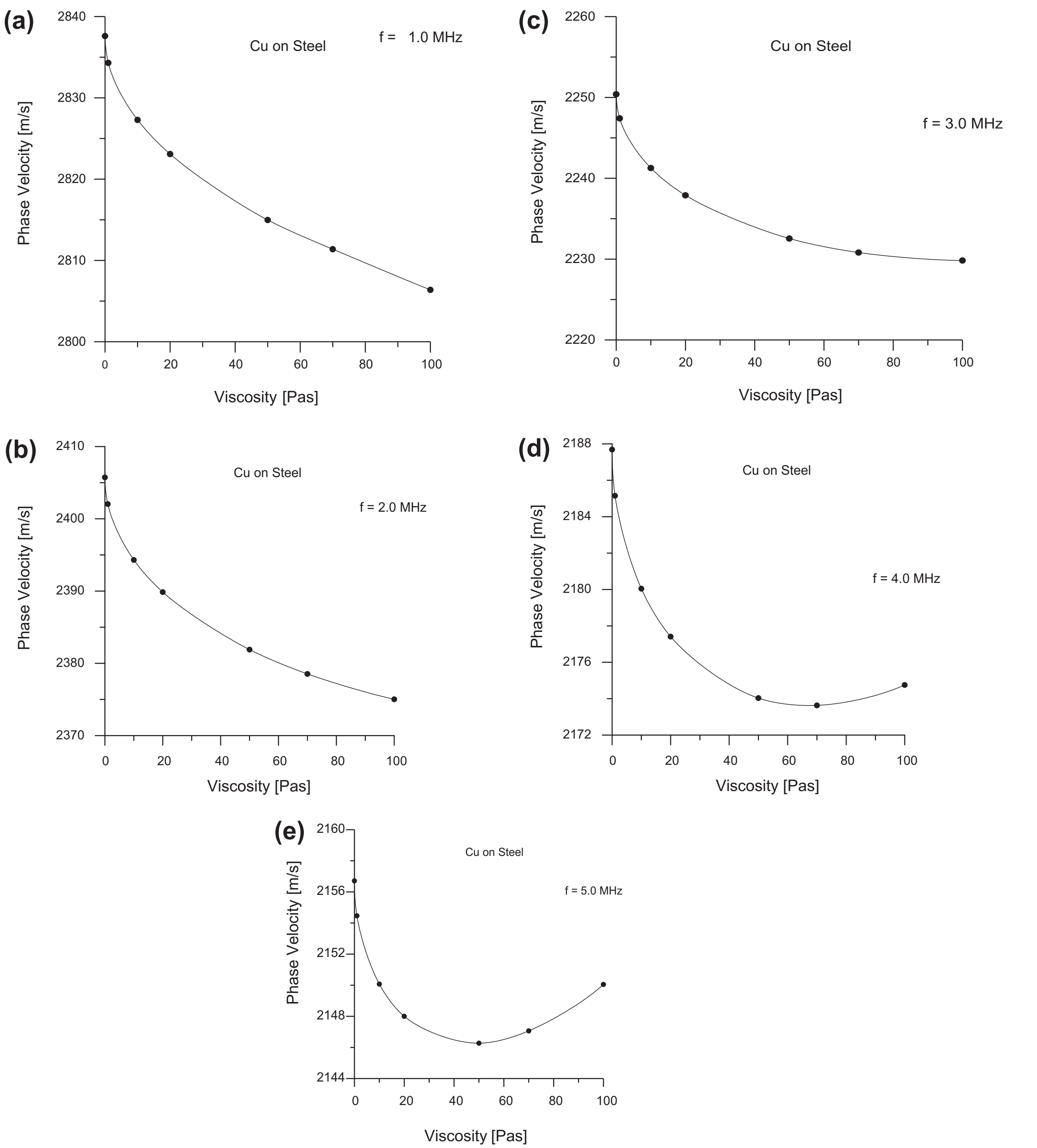

Fig. 2. Phase velocity dispersion curves in function of liquid viscosity for various values of frequency. (a) f = 1 MHz, (b) f = 2 MHz, (c) f = 3 MHz, (d) f = 4 MHz, (e) f = 5 MHz.

$$\lambda_1 = \left(k^2 - j\omega\frac{\rho_l}{\eta}\right)^{1/2} \quad \text{and} \quad Re(\lambda_1) > 0$$

C_4 is an arbitrary constant.

The condition $Re(\lambda_1) > 0$ assures that the Love wave amplitude in a liquid decays to zero with increasing distance from the waveguide surface $x_2 \to -\infty$.

The shear stress component needed in the boundary condition is given by:

$$\tau_{23}^{(l)} = \eta\frac{\partial v_3}{\partial x_2} = C_4 \cdot \eta \cdot \lambda_1 \cdot exp(\lambda_1 \cdot x_2) \cdot exp[j(kx_1 - \omega t)] \quad (16)$$

2.4. Boundary conditions

At the interface between the elastic surface layer and the substrate and the interface between the surface of the layer and a liquid, the mechanical displacement u_3 and the shear stress τ_{23} have to fulfill the conditions of continuity, i.e:

1. Continuity of the displacement field u_3 and stress τ_{23} at the solid–liquid interface ($x_2 = -D$):

$$\left.\frac{\partial u_3^{(1)}}{\partial t}\right|_{x_2=-D} = v_3|_{x_2=-D} \quad (17)$$

$$\left.\tau_{23}^{(1)}\right|_{x_2=-D} = \left.\tau_{23}^{(l)}\right|_{x_2=-D} \quad (18)$$

2. Continuity of the displacement field u_3 and stress τ_{23} at the interface between the layer and the substrate ($x_2 = 0$):

$$\left.u_3^{(1)}\right|_{x_2=0} = \left.u_3^{(2)}\right|_{x_2=0} \quad (19)$$

$$\left.\tau_{23}^{(1)}\right|_{x_2=0} = \left.\tau_{23}^{(2)}\right|_{x_2=0} \quad (20)$$

2.5. Dispersion relations

Substitution of (5), (7)–(9), (11)–(13), (15) and (16) into (17)–(20) results in the set of four linear and homogeneous equations for coefficients C_1, C_2, C_3 and C_4. For nontrivial solution, the determinant of this set of equations has to equal zero. This leads to the following (complex) dispersion relation:

$$\sin(qD) \cdot \{(\mu_1)^2 \cdot q^2 + \mu_2 \cdot b \cdot \lambda_1 \cdot j\omega\eta\} - \cos(qD) \cdot \{\mu_1 \cdot \mu_2 \cdot b \cdot q - \mu_1 \cdot q \cdot \lambda_1 \cdot j\omega\eta\} = 0 \quad (21)$$

Quantities q, b and λ_1 in Eq. (21) are complex.

Eq. (21) is complex dispersion equation of Love waves propagating in an elastic layered waveguide loaded on the surface with a viscous Newtonian liquid.

After separating the real and imaginary parts of Eq. (21) we obtain the following system of nonlinear algebraic equations (see Appendix):

$$A(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega;\ k_0, \alpha) = 0 \quad (22)$$

$$B(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega;\ k_0, \alpha) = 0 \quad (23)$$

Eqs. (22) and (23) constitute a system of nonlinear algebraic equations with unknowns k_0 and α.

The system of nonlinear Eqs. (22) and (23) was solved using the Newton method with a computer package Scilab. This method for the efficient solution of Eqs. (22) and (23) requires an appropriate choice of initial approximations. After finding a solution (k_0, α), one can calculate the phase velocity of the Love wave $v = \omega/k_0$. Imaginary part α of the wavenumber k represents the attenuation of the Love wave per unit length in the direction of propagation.

3. Numerical results

Numerical calculations were performed on the example of the Love wave waveguide structure composed of an isotropic, homogeneous, Cu surface layer deposited on an isotropic, homogeneous steel substrate. The surface of the Cu layer is in contact with a viscous liquid half-space ($x_2 < -D$). In the numerical computations the following values of parameters were assumed:

For Cu	For steel
$\mu_1 = 3.91 \cdot 10^{10}$ N/m^2	$\mu_2 = 8.02 \cdot 10^{10}$ N/m^2
$\rho_1 = 8.9 \cdot 10^3$ kg/m^3	$\rho_2 = 7.8 \cdot 10^3$ kg/m^3
$v_1 = (\mu_1/\rho_1)^{1/2} = 2096$ m/s	$v_2 = (\mu_2/\rho_2)^{1/2} = 3206.5$ m/s

The thickness D of the surface layer was 0.4 mm. The density of the liquid was assumed as $\rho_l = 1 \cdot 10^3$ kg/m^3.

Losses in the Cu layer and steel substrate were neglected. The only source of losses is the viscosity of the liquid.

Numerical analysis was performed in the frequency range from 0.5 to 5 MHz, and for values of viscosity from 0.1 to 100 Pas.

3.1. Phase velocity versus viscosity (f = const).

Fig. 2a–e show the effect of viscosity of a viscous liquid on the Love wave phase velocity for various values of frequency (f = 1,2,3,4 and 5 MHz).

It can be seen from Fig. 2a–e that with the increase of viscosity, the phase velocity decreases. Only for higher values of frequency (f = 4 and 5 MHz) the phase velocity of the Love wave initially decreases and then starts to augment with an increase in viscosity of the liquid (Fig. 2d and e).

A similar phenomenon was observed by (Guo and Sun, 2008) in the case of Bleustein–Gulyaev waves propagating in a waveguide loaded with a viscous liquid. Bleustein–Gulyaev waves are also SH surface acoustic waves that propagate in the metalized piezoelectric semi-space. Possible physical explanation for this phenomenon may be that a high viscosity liquid loading the

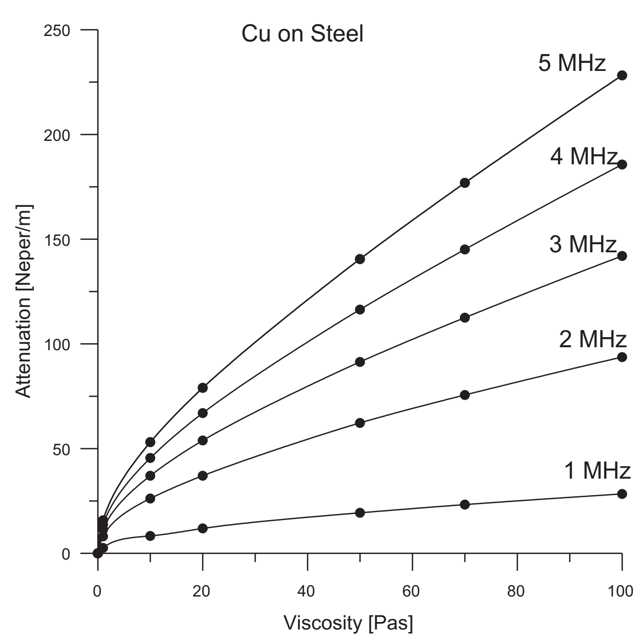

Fig. 3. Dispersion curves of the attenuation in function of liquid viscosity for various values of frequency.

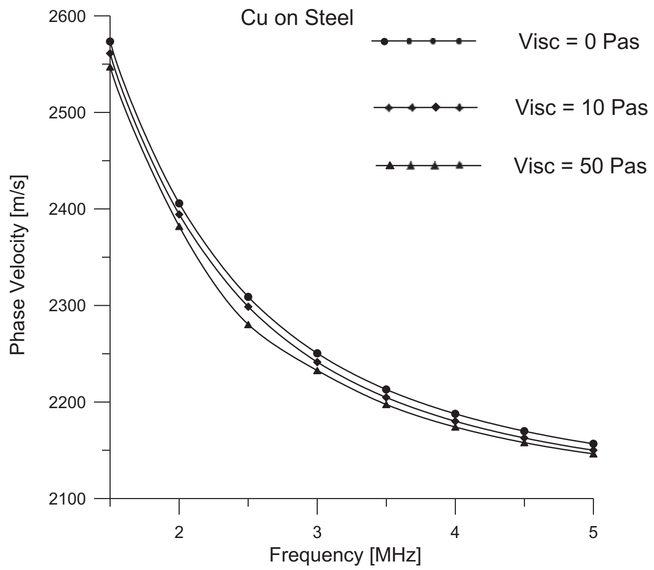

Fig. 4. Phase velocity dispersion curves in function of frequency for various values of liquid viscosity.

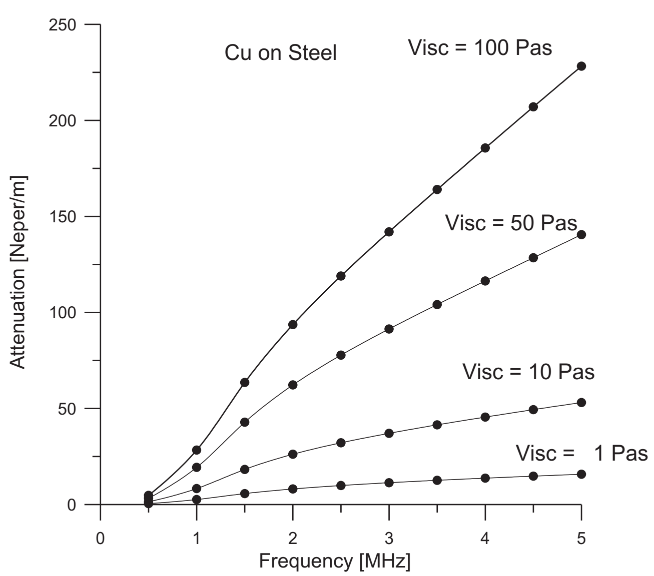

Fig. 5. Dispersion curves of the attenuation in function of frequency for various values of liquid viscosity.

waveguide surface stiffens the material in the surface layer. This can cause an increase in the phase velocity of the Love wave.

3.2. Attenuation versus viscosity (f = const.)

Fig. 3 shows changes of attenuation in function of liquid viscosity at various values of frequency (f = 1,2,3,4 and 5 MHz).

It is seen from Fig. 3 that wave attenuation increases monotonically with the increase of liquid viscosity.

3.3. Phase velocity versus frequency (η = const.)

Fig. 4 illustrates the influence of the viscosity of liquid on the dispersion curves of phase velocity for viscosities η = 0, 10 and 50 Pas.

Fig. 4 indicates that the phase velocity decreases with the increase of liquid viscosity.

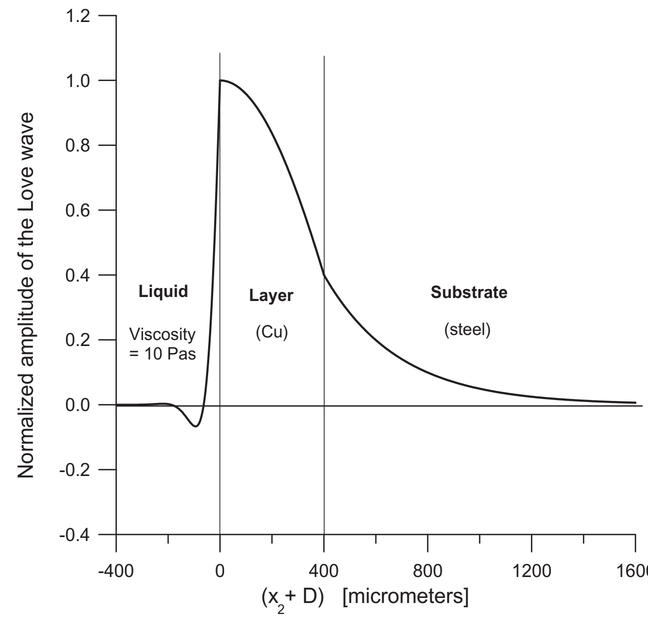

Fig. 6. The real part of the normalized amplitude of the mechanical displacement field of the Love wave propagating in the elastic Cu on steel waveguide that is loaded on the surface by a viscous liquid, η = 10 Pas.

3.4. Attenuation versus frequency (η = const.)

The dispersion curves of the attenuation versus frequency were also calculated and plotted as shown in Fig. 5.

Fig. 5 shows that the attenuation α of the Love wave is a monotonic function of the frequency.

3.5. Love wave amplitude distribution

Variation with depth x_2 of the mechanical displacement of the Love wave propagating in the Cu on steel elastic waveguide loaded on the surface with a viscous liquid semi-space is shown in Fig. 6.

In the viscous liquid region variations of the mechanical displacement with depth exhibit oscillatory behavior.

For η = 10 Pas and f = 2 MHz, the penetration depth equals $\delta = 40\mu$m.

The presence of a viscous liquid has little effect on the mechanical displacement distribution in the elastic surface layer and the substrate.

4. Conclusions

The propagation of Love waves in the elastic layered waveguide is investigated analytically and numerically. The surface of the waveguide is loaded with a viscous (Newtonian) liquid. The presence of a viscous liquid on the waveguide surface introduces losses and the Love wave exhibits attenuation as it travels. The occurrence of losses causes that the mathematical analysis of the problem is much more complicated.

Analytical form of the complex dispersion equation was obtained. After separating the real and imaginary parts of the dispersion equation, the resulting system of nonlinear algebraic equations was numerically solved. As a result, the graphs of phase velocity on frequency (for a given viscosity) and the phase velocity curves as a function of liquid viscosity at a fixed values of frequency were obtained. Moreover, the wave attenuation curves of Love waves versus the liquid viscosity and frequency were obtained. It was found that the increase in viscosity reduces the phase velocity and augments the attenuation of the wave.

Love wave amplitude distribution as a function of depth x_2, for liquid viscosity of 10 Pas has also been determined. It was stated that for a viscous liquid loading, the amplitude of the Love wave varies with depth in an oscillatory way, and decays to zero for $x_2 \to \pm\infty$.

The results of this work are fundamental and can be applied in the design of liquid viscosity sensors and biosensors, in geophysics, seismology and in the NDT of materials.

Appendix A

In the dispersion equation (Eq. (21)), the quantities q, b and λ_1 are complex:

$$q^2 = \left(k_1^2 - k_0^2 + \alpha^2\right) - j \cdot 2 \cdot k_0 \cdot \alpha \tag{A.1}$$

$$b^2 = \left(k_0^2 - \alpha^2 - k_2^2\right) + j \cdot 2 \cdot k_0 \cdot \alpha \tag{A.2}$$

$$\lambda_1^2 = \left(k_0^2 - \alpha^2\right) - j \cdot \left(\omega \cdot \frac{\rho_l}{\eta} - 2 \cdot k_0 \cdot \alpha\right) \tag{A.3}$$

where

$$k_1 = \frac{\omega}{v_1}; \quad k_2 = \frac{\omega}{v_2}; \quad k_0 = \frac{\omega}{v}$$

For typical values of frequency (of the order of several MHz) and viscosity (up to 100 Pas) the second terms in (A.1) and (A.2) are much smaller than the corresponding first components. Similarly, the first term in (A.3) is much smaller than the second term. Consequently, using these relations we can write:

$$q = \sqrt{\left(k_1^2 - k_0^2 + \alpha^2\right)} - j \frac{k_0 \cdot \alpha}{\sqrt{\left(k_1^2 - k_0^2 + \alpha^2\right)}} = c + jd \tag{A.4}$$

$$b = \sqrt{\left(k_0^2 - \alpha^2 - k_2^2\right)} + j \frac{k_0 \cdot \alpha}{\sqrt{\left(k_0^2 - \alpha^2 - k_2^2\right)}} = e + jf \tag{A.5}$$

$$\begin{aligned}\lambda_1 &= \frac{1}{2 \cdot \sqrt{2}} \cdot \frac{\left(k_0^2 - \alpha^2\right)}{\sqrt{(\omega \cdot \frac{\rho_l}{\eta} - 2 \cdot k_0 \cdot \alpha)}} + \sqrt{\frac{(\omega \cdot \frac{\rho_l}{\eta} - 2 \cdot k_0 \cdot \alpha)}{2}} \\ &\quad + j \cdot \left\{\frac{1}{2 \cdot \sqrt{2}} \cdot \frac{\left(k_0^2 - \alpha^2\right)}{\sqrt{\left(\omega \cdot \frac{\rho_l}{\eta} - 2 \cdot k_0 \cdot \alpha\right)}} - \sqrt{\frac{\left(\omega \cdot \frac{\rho_l}{\eta} - 2 \cdot k_0 \cdot \alpha\right)}{2}}\right\} \\ &= a_1 + jb_1\end{aligned} \tag{A.6}$$

Substituting equations (A.4)–(A.6) to the dispersion equation (Eq. (21)) and grouping the real and imaginary terms we obtain:

$$(y_1 \cdot y_3 - y_2 \cdot y_4) - (y_5 \cdot y_7 + y_6 \cdot y_8) = 0 \tag{A.7}$$

$$(y_1 \cdot y_4 - y_2 \cdot y_3) - (y_5 \cdot y_8 + y_6 \cdot y_7) = 0 \tag{A.8}$$

where

$$y_1 = \sin(c \cdot D) \cdot \cosh(d \cdot D) \tag{A.9}$$

$$y_2 = \cos(c \cdot D) \cdot \sinh(d \cdot D) \tag{A.10}$$

$$y_3 = (\mu_1)^2 \cdot (c \cdot c - d \cdot d) - \omega \cdot \eta \cdot \mu_2 \cdot (f \cdot a_1 + e \cdot b_1) \tag{A.11}$$

$$y_4 = (\mu_1)^2 \cdot 2 \cdot c \cdot d - \omega \cdot \eta \cdot \mu_2 \cdot (e \cdot a_1 - f \cdot b_1) \tag{A.12}$$

$$y_5 = \cos(c \cdot D) \cdot \cosh(d \cdot D) \tag{A.13}$$

$$y_6 = \sin(c \cdot D) \cdot \sinh(d \cdot D) \tag{A.14}$$

$$y_7 = \mu_2 \cdot \mu_1 \cdot (e \cdot c - f \cdot d) - \omega \cdot \eta \cdot \mu_1 \cdot (b_1 \cdot c - d \cdot a_1) \tag{A.15}$$

$$y_8 = \mu_2 \cdot \mu_1 \cdot (f \cdot c + d \cdot e) - \omega \cdot \eta \cdot \mu_1 \cdot (a_1 \cdot c - b_1 \cdot d) \tag{A.16}$$

The dependencies (A.7) and (A.8) can be written in the form:

$$A(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega;\ k_0, \alpha) = 0 \tag{A.17}$$

$$B(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega;\ k_0, \alpha) = 0 \tag{A.18}$$

This is a system of nonlinear algebraic equations. The unknowns are: k_0 and α.

The parameters are: μ_1, ρ_1, μ_2, ρ_2, η, ρ_l, D and ω.

Equations (A.17) and (A.18) constitute dispersion relations describing Love wave propagation in the waveguide loaded on the surface with a viscous Newtonian liquid.

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