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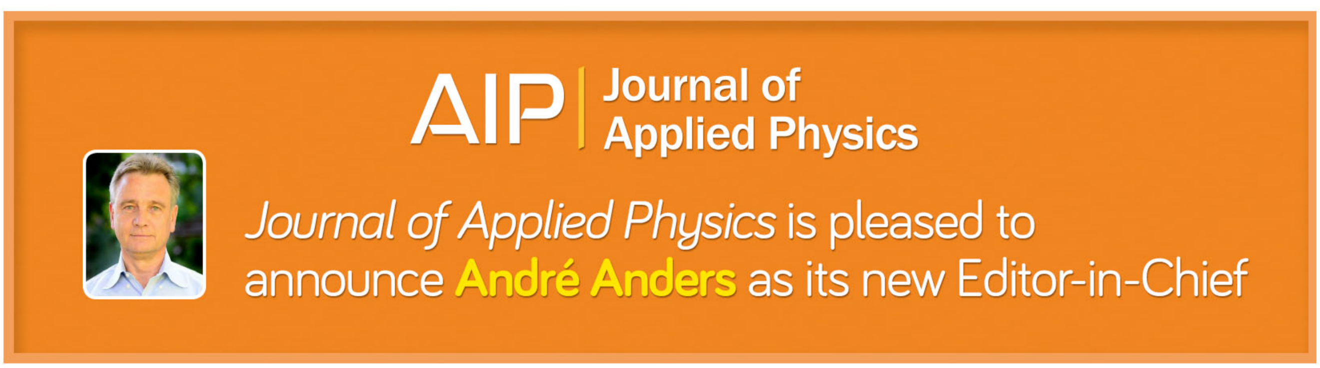

Inverse procedure for simultaneous evaluation of viscosity and density of Newtonian liquids from dispersion curves of Love waves

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Simultaneous determination of the viscosity and density of liquids is of great importance in the monitoring of technological processes in the chemical, petroleum, and pharmaceutical industry, as well as in geophysics. In this paper, the authors present the application of Love waves for simultaneous inverse determination of the viscosity and density of liquids. The inversion procedure is based on measurements of the dispersion curves of phase velocity and attenuation of ultrasonic Love waves. The direct problem of the Love wave propagation in a layered waveguide covered by a viscous liquid was formulated and solved. Love waves propagate in an elastic layered waveguide covered on its surface with a viscous (Newtonian) liquid. The inverse problem is formulated as an optimization problem with appropriately constructed objective function that depends on the material properties of an elastic waveguide of the Love wave, material parameters of a liquid (i.e., viscosity and density), and the experimental data. The results of numerical calculations show that Love waves can be efficiently applied to determine simultaneously the physical properties of liquids (i.e., viscosity and density). Sensors based on this method can be very attractive for industrial applications to monitor on-line the parameters (density and viscosity) of process liquid during the course of technological processes, e.g., in polymer industry. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4891018]

I. INTRODUCTION

Simultaneous determination of liquid viscosity and density is of great importance in monitoring technological parameters of liquids during the course of industrial processes as well as in geophysics. The continuous on-line measurements of key physical parameters of the process liquid, such as viscosity and density, are important for quality control and for enhancing productivity.

Traditional mechanical methods for characterization of liquid viscosity and density are cumbersome, time consuming, and difficult to computerize. Hence, due to their inherent limitations, these mechanical methods cannot be applied on-line to control technological processes.

Lately, to overcome the disadvantages of the mechanical methods, new ultrasonic methods that use ultrasonic bulk shear waves,[1] and surface acoustic waves (SAW), e.g., that of the Love type were introduced to investigate the physical parameters of liquids.[2–8] Furthermore, to evaluate the mechanical properties of liquids, the ultrasonic waves of quasi-longitudinal type were applied.[9–11] However, their use in liquid properties sensors is limited due to their lower sensitivity.

SAWs can be classified into two general groups: Rayleigh-type waves and shear horizontal (SH) waves. The former waves have at least two components of vibrations, i.e., longitudinal and vertical transverse, which cannot be separated. When Rayleigh waves propagate at a solid–liquid interface, the surface normal displacement radiates compressional waves into the liquid. Consequently, Rayleigh waves can be completely attenuated within the propagation range of the sensing device. Therefore, Rayleigh waves are impractical for use in the measurements of liquid viscosity.

By contrast, the Love wave is SH surface acoustic wave that has only one component of the mechanical displacement. Ultrasonic Love waves propagate in an elastic layered waveguide consisting of an elastic surface layer deposited rigidly on an elastic substrate. In this study, we consider the case when the waveguide surface is covered by a viscous (Newtonian) liquid. Dispersion curves (phase velocity and attenuation versus frequency) of ultrasonic Love waves propagating in such structure depend on the material parameters of the layered waveguide structure and are sensitive to changes in the mechanical parameters of the loading liquid (e.g., viscosity and density).[12–16] The sensitivity of the dispersion curves on changes in viscosity and density of the liquid constitutes the physical basis of a new, proposed by the authors, Inverse Method for the simultaneous determination of viscosity and density of liquids.

In this paper, the authors present a new inverse method (based on the use of the Love surface wave) that allows the simultaneous determination of the key physical parameters of liquids, such as viscosity and density. The established inverse procedure employs the measured dispersion curves of phase velocity and attenuation of Love waves that propagate in an elastic layered waveguide covered on its surface by a viscous (Newtonian) liquid. This new inverse method could constitute the basis for the design and construction of multimeasurand liquid sensors of the physical properties of liquids.

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To date, liquid viscosity and density were measured simultaneously using ultrasonic dual element sensors[17–19] or special highly complex sensors applicable only in laboratory conditions.[20–22] In the methods using dual element sensors, two sensing elements (waveguides or resonators) are necessary. Developed by the authors, an original inverse method that uses the Love wave is superior in relation to the existing so far ultrasonic methods for the simultaneous evaluation of liquid viscosity and density. In contrast to those methods, our method is more simple and uses only one single miniaturized sensing element (i.e., Love wave waveguide).

Evaluation of Love wave parameters (i.e., phase velocity, attenuation and distribution on depth of the mechanical displacement) for known "a priori" values of mechanical parameters of an elastic waveguide and liquid constitute a direct problem. The direct problem for Love waves was formulated and solved in the authors' previous paper.[15]

The inverse problem relies on determination of the material parameters (e.g., the unknown values of the viscosity and density of a liquid) from measurements of dispersion curves (phase velocity and attenuation as a function of frequency) for Love waves. In this paper, the inverse problem was formulated and solved as a minimization problem[23] with properly defined objective functions. The objective function that depends on the unknown liquid viscosity and density, and known "a priori" the material parameters of the waveguide structure, the frequency, and experimental data (phase velocity and attenuation of the surface Love wave) has been constructed.

The experimental dispersion curves were derived by adding random noise to the exact dispersion curves. To minimize the considered objective function, Nelder-Mead optimization procedures from Scilab package were employed. It enabled the determination of the optimum values of liquid viscosity and density simultaneously. These optimum values of liquid viscosity and density constitute the solution of the inverse problem.

Numerical calculations were performed for the following waveguide structure: Cu (copper) surface layer on steel substrate, loaded on its surface with a viscous (Newtonian) liquid.

The direct problem of the Love wave propagation in the waveguides covered with a viscous liquid has been solved in the authors' work.[15] The results of this work are shortly recalled in Sec. II. The inverse problem along with appropriately constructed objective function are presented in Sec. III. Section IV describes the dispersion curves of phase velocity and attenuation obtained in the numerical experiment (synthetic data). Results of numerical calculations are presented in Sec. V. Discussion is inserted in Sec. VI. Section VII contains conclusions.

II. DIRECT PROBLEM

Calculation of the dispersion curves and the mechanical displacement of the surface wave for given values of liquid viscosity and density, elastic parameters of the surface layer and substrate, and frequency, forms a direct Sturm-Liouville problem. The Love wave propagates in a layered elastic waveguide (loss-less elastic isotropic layer is rigidly attached to a loss-less isotropic and elastic substrate). The top surface of the layer is loaded with a viscous (Newtonian) liquid, see Fig. 1. The above mentioned direct problem for Love waves propagating in the structure shown in Fig. 1 was formulated and solved in the authors' previous paper.[15] In this section, we present in brief the main results of this study.

The mechanical displacement of the Love wave u_3 is polarized along the x_3 axis, perpendicular to the direction of propagation x_1. Consequently, we can write: $u_3(x_1, x_2, \mathrm{t}) = \mathrm{f}(x_2) \times \exp[\mathrm{j}(kx_1 - \omega \mathrm{t})]$, where $\mathrm{f}(x_2)$ describes the dependence of the Love wave amplitude on the depth x_2, k is the complex wave number, ω is the angular frequency, and t is time. The waveguide surface is at $x_2 = -D$. The considered problem is two-dimensional, having no variation along the x_3 axis.

The Love wave exhibits a multimode character. In many practical applications (e.g., in sensors and NDT), the most important is the fundamental mode of Love waves. Therefore, in this study, we have restricted our attention to the propagation of the fundamental (the lowest) mode of Love waves.

The function $f(x_2)$ in the region of the substrate ($x_2 > 0$), the surface layer ($-D < x_2 < 0$), and in the liquid ($x_2 < -D$) satisfies the appropriate differential equation resulting from the equations of motion. After solving these equations, we obtain analytic expressions for the mechanical displacement u_3 of the wave and shear stress τ_{23} in each of these regions, respectively.

The details of the theory of the Love wave propagation in an elastic waveguide covered on its surface with a viscous liquid are presented in Ref. 15.

A. Dispersion equation

Using the continuity conditions of the mechanical displacement u_3 and shear stress τ_{23} at the interfaces: (1) $x_2 = 0$ and (2) $x_2 = -D$, and equating to zero the determinant of the resultant matrix equation for the unknown coefficients C_1, C_2, C_3, and C_4, we arrive at the following analytical expression for the complex dispersion equation of the Love wave:[15]

$$\sin(qD) \times \{(\mu_1)^2 \times q^2 + \mu_2 \times b \times \lambda_1 \times j\omega\eta\} + \\ -\cos(qD) \times \{\mu_1 \times \mu_2 \times b \times q - \mu_1 \times q \times \lambda_1 \times j\omega\eta\} = 0. \quad (1)$$

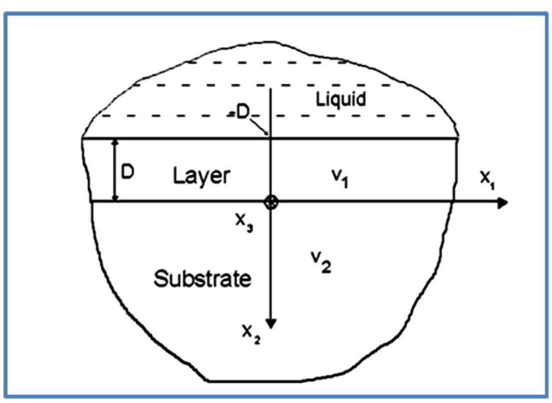

FIG. 1. Geometry of a Love wave waveguide ($v_1 < v_2$) covered by a viscous (Newtonian) liquid.

In Eq. (1), the complex wave number equals to $k = k_0 + j\alpha$, where $j = (-1)^{1/2}$.

The real part of the wave number k_0 determines the phase velocity of the Love wave. The imaginary part of the wave number α is an attenuation of the Love wave.

In the dispersion equation (Eq. (1)), the quantities q, b, and λ_1 are complex

$$q^2 = (k_1^2 - k_0^2 + \alpha^2) - j \times 2 \times k_0 \times \alpha \tag{2}$$

$$b^2 = (k_0^2 - \alpha^2 - k_2^2) + j \times 2 \times k_0 \times \alpha \tag{3}$$

$$\lambda_1^2 = (k_0^2 - \alpha^2) - j \times \left(\omega \times \frac{\rho_l}{\eta} - 2 \times k_0 \times \alpha\right), \tag{4}$$

where $k_1 = \frac{\omega}{v_1}$; $k_2 = \frac{\omega}{v_2}$; $k_0 = \frac{\omega}{v}$, ρ_l is the liquid density, $v_1 = (\mu_1/\rho_1)^{1/2}$ is the bulk shear wave velocity in the layer, $v_2 = (\mu_2/\rho_2)^{1/2}$ is the bulk shear wave velocity in the substrate, μ_1, μ_2 and ρ_1, ρ_2 correspond, respectively, to the shear modulus and mass density in the surface layer (index 1) and in the substrate (index 2), v is the Love wave phase velocity, η is liquid viscosity, and ω is the angular frequency.

Equation (1) is complex dispersion equation of Love waves propagating in an elastic layered waveguide loaded on the surface with a viscous (Newtonian) liquid.

After separating the real and imaginary parts of Eq. (1), we obtain the following system of nonlinear algebraic equations with unknowns: k_0 and α[15]

$$A(\mu_1,\ \rho_1,\ \mu_2,\ \rho_2, \eta,\ \rho_l, D, \omega;\ k_0,\ \alpha) = 0 \tag{5}$$

$$B(\mu_1,\ \rho_1,\ \mu_2,\ \rho_2, \eta,\ \rho_l, D, \omega;\ k_0,\ \alpha) = 0. \tag{6}$$

Here, μ_1, ρ_1, μ_2, ρ_2, η, ρ_l, D and ω are parameters. The unknowns are k_0 and α.

Analytical form of nonlinear functions A and B is given by Eqs. (A7) and (A8) in the work of authors.[15]

The system of nonlinear equations (5) and (6) was solved using the appropriate numerical procedures from a computer package Scilab.

After finding a solution (k_0, α), one can calculate the phase velocity of the Love wave $v = \omega/k_0$. Imaginary part α of the complex wavenumber k represents the attenuation of the Love wave per unit length in the direction of propagation. Consequently, the dispersion curves of Love waves are evaluated.

The results obtained from the solution of the Direct Problem will be employed in the numerical simulations of the experimental data (synthetic data), as well as in solving the Inverse Problem.

B. Sensitivity analysis

It is important to find out which material parameters of a liquid and the elastic waveguide have a noticeable influence on the phase velocity and attenuation of the Love wave. Therefore, we present a sensitivity analysis.

Solving the dispersion equations Eqs. (5) and (6) (Direct Problem), curves of phase velocity and attenuation of the Love wave as a function of the viscosity and density of a liquid covering the surface of the waveguide have been obtained, see Figs. 2 and 3.

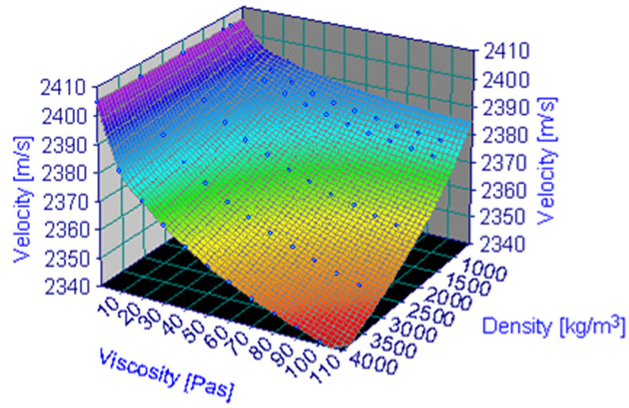

FIG. 2. Variation of phase velocity of the Love wave with density and viscosity of a viscous liquid, f = 2 MHz.

The material parameters of the Love wave waveguide have been assumed the same as in the numerical calculations in Sec. V.

As can be seen from Figs. 2 and 3, both the density and viscosity of the liquid are influential on the phase velocity and attenuation of the Love wave. Attenuation and velocity of the Love wave are sensitive to changes in density and viscosity of the liquid loading surface of the waveguide. This fact was exploited in the construction of the objective function $\Pi(\eta, \rho_l)$, in the Inverse Method in Sec. III.

The conducted numerical calculations have also shown that the velocity and attenuation dispersion curves of the Love wave are less dependent on changes of material parameters and the geometry of the waveguide.

III. INVERSE PROBLEM

The inverse problem relies on the determination of the unknown liquid parameters from the measured dispersion curves of phase velocity and attenuation of Love waves propagating in a considered layered structure. In this study, the inverse problem was formulated and solved as an optimization problem with properly defined objective function.

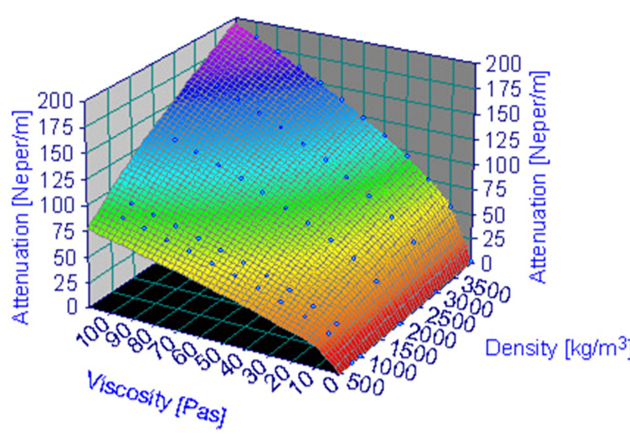

FIG. 3. Variation of attenuation of the Love wave with density and viscosity of a viscous liquid, f = 2 MHz.

To solve the inverse problem, the following three steps were performed

1. formulation and solution of the direct problem
2. performing numerical experiment (synthetic data)
3. formulation and solution of the inverse procedure
   a. inverse problem is formulated as an optimization problem
   b. determination of objective function $\Pi(\eta,\ \rho_l)$
   c. application of optimization procedures

$$\min \Pi(\eta,\ \rho_l)\ \Rightarrow (\eta_{opt},\ (\rho_l)_{opt}).$$

Formulation and solution of the direct problem is presented in Sec. II.

A. Objective function

An objective function $\Pi(\eta,\ \rho_l)$ depending on liquid viscosity and density, materials parameters of the structure, frequency, and experimental data (phase velocity and attenuation of the Love wave) was introduced and defined.

The objective function $\Pi(\eta,\ \rho_l)$ (see Eq. (8)) is employed in the inverse problem that relies on simultaneous determination of two unknown operational variables, i.e., liquid viscosity η and density ρ_l.

$$\Pi(\eta,\rho_l) = \sum_{j=1}^{N_e}\left\{\left(\frac{v_j^m - v_j^c(\eta,\ \rho_l)}{v_j^m}\right)^2 + \left(\frac{\propto_j^m - \propto_j^c(\eta,\ \rho_l)}{\propto_j^m}\right)^2\right\}. \tag{7}$$

Here N_e is the number of experimental points, v_j^c and $\propto_j^c$ are the phase velocity and attenuation calculated from the direct problem that depend on both unknown values of liquid viscosity η and liquid density ρ_l, known "a priori": material parameters of an elastic waveguide, and experimental angular frequency ω_j. v_j^m and $\propto_j^m$ are experimental quantities of the phase velocity and attenuation evaluated in Sec. IV.

Making use of the optimization methods, minimum of the objective function $\Pi(\eta,\ \rho_l)$ was evaluated. It enabled the determination of the optimum values of liquid viscosity η and density ρ_l simultaneously. These optimum values of liquid viscosity and density constitute the solution of the inverse problem.

To minimize the considered objective function, optimization procedures of the Nelder-Mead type from Scilab software package were employed. In these Scilab optimization routines, guess initial values for the unknowns (η *and* ρ_l) are required. The above mentioned numerical procedures are especially convenient to manage objective functions disturbed by random errors. Nelder-Mead algorithm (or downhill simplex method) is a direct search optimization procedure (in which derivatives of the objective function may not be known). Moreover, Nelder-Mead algorithm was effectively used to solve Inverse Problems that require multiple solving of the Direct Problem (direct Sturm-Liouville Problem).[24] It is also well suited for solving problems with a small number of unknowns (<20). In order to verify the results of the optimization process, another optimization algorithm (from Scilab software package) was used, i.e., simulated annealing algorithm. Simulated Annealing (SA) is a random-search optimization procedure which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system. The major advantage of the Simulated Annealing algorithm over other methods is that it allows to reach the global minimum. The results obtained from the use of both methods were consistent; however, calculations performed by using the annealing algorithm are much more time consuming.

IV. NUMERICAL EXPERIMENT (SYNTHETIC DATA)

The dispersion curves were evaluated for the Love wave propagating in the layered elastic waveguide structure loaded on its surface with a viscous liquid, see Fig. 1.

In our numerical experiments, we determined the dispersion curves numerically. In the first step, by solving the direct problem (Eqs. (5) and (6)), we calculated the phase velocity and attenuation curves for the exact values of liquid viscosity $\eta^{\text{exact}} = 1\,\text{Pa·s}$ and density $\rho_l^{\text{exact}} = 1 \times 10^3\,\text{kg/m}^3$, see Figs. 4, 5, and Table I. Used in the numerical calculations material parameters of the waveguide structure (Cu on steel) are given in Sec. V.

The dispersion curves represented in Figs. 4 and 5 by solid lines (not corrupted by noise) are regarded as exact dispersion curves.

In the Inverse Method for the simultaneous measurement of the viscosity and density a liquid, broadband measurement of the dispersion curves of Love wave velocity and attenuation is required. Effect of frequency bandwidth on the results obtained was investigated using in the Inverse Method dispersion curves for the Love wave evaluated for (a) 4, (b) 6, and (c) 10 frequencies. No significant changes in the accuracy of the results obtained from the Inverse Method for these three cases were not stated. Therefore, in this work

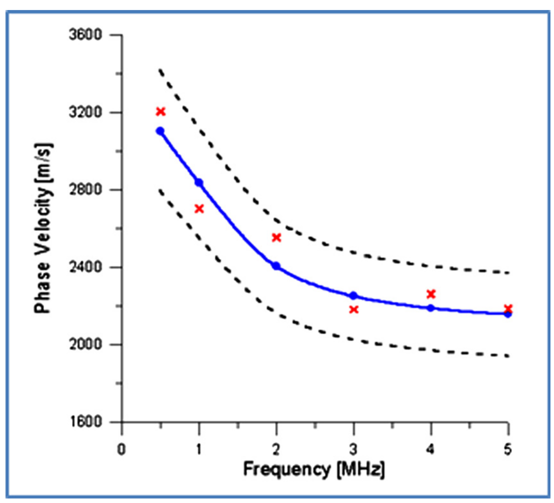

FIG. 4. Dispersion curves of phase velocity of the Love wave propagating in the layered waveguide presented in Fig. 1, $\eta^{\text{exact}} = 1\,\text{Pa·s}$ and $\rho_l^{\text{exact}} = 1 \times 10^3\,\text{kg/m}^3$. Upper and lower dashed curves delimit the dispersion curves corrupted by 10% maximum random error. Exemplary random realization of the corrupted dispersion curve of phase velocity was marked by crosses (x).

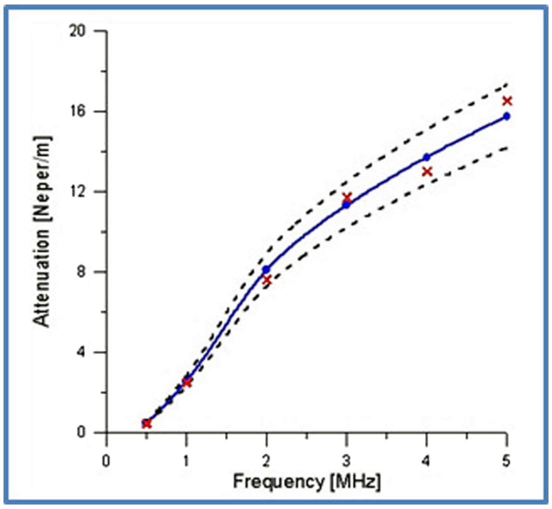

FIG. 5. Dispersion curves of attenuation of the Love wave propagating in the layered waveguide presented in Fig. 1, $\eta^{\mathrm{exact}} = 1\,\mathrm{Pa \cdot s}$ and $\rho_l^{\mathrm{exact}} = 1 \times 10^3\,\mathrm{kg/m^3}$. Upper and lower dashed curves delimit the dispersion curves corrupted by 10% maximum random error. Exemplary random realization of the corrupted dispersion curve of attenuation was marked by crosses (x).

the authors used in the Inverse Method velocity and attenuation dispersion curves of the Love wave evaluated for 6 various measurement frequencies.

Subsequently, we added random errors (e.g., 1%, 5%, 10%) to these values of the phase velocity and attenuation (columns 2 and 3 in Table I). Obtained in this way dispersion curves are treated in the inverse procedure as simulated experimental curves. Dispersion curves were calculated and disturbed by random noise for 6 various values of frequency: $f = 0.5\,\mathrm{MHz}$, 1.0 MHz, 2.0 MHz, 3.0 MHz, 4.0 MHz, and 5.0 MHz.

Experimental bandwidth is selected in the range of the highest sensitivity (to changes of parameters of the liquid), of the considered sensor based on the Love wave.

By consecutive use of a random number generator (with Scilab software package), we have produced a set of noise distorted dispersion curves. These dispersion curves are treated as experimental dispersion curves (synthetic data). These curves are then used in the Inverse Method for calculating the unknown values of viscosity and density (η, ρ_l) of a viscous liquid.

V. RESULTS OF NUMERICAL CALCULATIONS

Numerical calculations were carried out for the waveguide structure: Cu (copper) surface layer deposited on the steel substrate. Surface of the Cu layer is loaded with a viscous liquid half-space ($x_2 < -D$).

In the numerical computations, the following values of material parameters were used:

For Cu	For steel
$\mu_1 = 3.91 \times 10^{10}\,\mathrm{Pa}$	$\mu_2 = 8.02 \times 10^{10}\,\mathrm{Pa}$
$\rho_1 = 8.9 \times 10^3\,\mathrm{kg/m^3}$	$\rho_2 = 7.8 \times 10^3\,\mathrm{kg/m^3}$
$v_1 = (\mu_1/\rho_1)^{1/2} = 2096\,\mathrm{m/s}$	$v_2 = (\mu_2/\rho_2)^{1/2} = 3206.5\,\mathrm{m/s}$

The thickness D of the surface layer is 0.4 mm. Losses in the Cu layer and steel substrate are neglected. The only source of losses is the viscosity of the liquid.

Minimization of the objective function $\Pi(\eta, \rho_l)$ provides simultaneously the unknown values of both the viscosity η and density ρ_l of a liquid, respectively. To minimize the considered objective function, Nelder-Mead (nonlinear simplex) optimization procedures from Scilab software package were employed.

The relative error for a series of N measurements of liquid viscosity was defined as follows:

$$Relative\,Error = \left\{ \left(\frac{|\eta_1^{calc} - \eta^{exact}|}{|\eta^{exact}|} + \frac{|\eta_2^{calc} - \eta^{exact}|}{|\eta^{exact}|} + \cdots + \frac{|\eta_{N_e}^{calc} - \eta^{exact}|}{|\eta^{exact}|} \right) \right\} \Big/ N. \tag{8}$$

An analogous formula holds also for the relative error for a series of N measurements of the density of a liquid.

We formulated and solved the inverse problem for the determination simultaneously two unknown variables, i.e., liquid viscosity, η, and density, ρ_l. The inverse problem was formulated as an optimization problem, where the objective function $\Pi(\eta, \rho_l)$, (Eq. (7)), is minimized. This inverse problem has been solved for several values of added (to exact values of the phase velocity and attenuation) random errors ranging from 0.1% to 10%, see Table II.

Rows 4 and 7 display errors of arithmetic average of 10 consecutive evaluations of liquid viscosity η, and density ρ_l. These errors are lower than the corresponding relative errors

TABLE I. Exact values of the phase velocity and attenuation at various frequencies for $\eta^{\mathrm{exact}} = 1\,\mathrm{Pa \cdot s}$, $\rho_l^{\mathrm{exact}} = 1 \times 10^3\,\mathrm{kg/m^3}$.

Frequency (MHz)	Phase velocity (m/s)	Attenuation (Neper/m)
0.5	3101.808	0.4332
1.0	2834.308	2.5681
2.0	2402.049	8.0815
3.0	2247.417	11.3111
4.0	2185.157	13.6913
5.0	2154.466	15.7249

TABLE II. Relative error of the simultaneous determination of both the viscosity and density of the liquid evaluated from the inverse method using objective function w $\Pi(\eta,\ \rho_l)$, for the maximum random errors equal to 0.1%, 1%, 5%, and 10%. Average value is the arithmetic mean of 10 consecutive evaluations of liquid viscosity and density respectively. Each evaluation of (η, ρ_l) results from the minimization of the objective function $\Pi(\eta, \rho_l)$, (Eq. (7)), for subsequent simulated dispersion curves of phase velocity and attenuation, see Sec. V).

Random error	0.1%	1%	5%	10%
Relative error (η) (%)	1.38	5.56	12.27	18.47
Average value (η)	0.99189	0.9694	0.9556	0.9516
Error (η) (%)	0.8108	3.0500	4.4364	4.8310
Relative error (ρ_l) (%)	1.41	5.92	13.96	19.94
Average value (ρ_l)	1008.34	1036.30	1060.71	1086.39
Error (ρ_l) (%)	0.8342	3.6300	6.0711	8.6394

inserted in rows 2 and 5. This is due to the fact that averaging reduces the level of random errors.

VI. DISCUSSION

The accuracy of the results (Table II) obtained from the solution of the Inverse Problem using the objective function $\Pi(\eta, \rho_l)$, for the dispersion curves corrupted by noise, is good. Table II shows that, for example, the 5% measurement accuracy of speed and wave attenuation gives relative error for the evaluation of viscosity equal to 12% and that for the density evaluation equal to 13% (column 4). Measurement accuracy can be enhanced by using the averaging process. In this case, after calculating the average of 10 measurements, the relative error of determining the viscosity decreased from 12% to 4%. Similarly, the relative error of determining the density was reduced from 13% to 6%. Practically, these errors can be even lower.

Accuracy of 6% (relative error) for the simultaneous measurement of liquid viscosity and density is good for industrial applications. This makes our inverse procedure attractive for on-line applications, in order to monitor the viscosity and density of the liquid during the course of technological processes.

Ultrasonic methods used so far for simultaneous measurement of fluid viscosity and density are highly complex,[20,25,26] and the sensors applied have a complicated structure. By contrast, our sensor (that is composed of a single Love wave waveguide) is simple in structure, robust, and can be used in harsh environment.

Referring to the other ultrasonic methods for simultaneous measurement of the viscosity and density of the liquid, it should be noted that many methods in fact determine the product of the density and viscosity $(\eta \cdot \rho_l)^{1/2}$.[21,27,28] Whereas, our method determines the unknown density and viscosity of the liquid independently.

Moreover, it can be noted that the accuracy of these other methods of determining the viscosity and density of liquids is limited. For example, in Ref. 28, density measurement accuracy achieved was of the order of 20%. In Ref. 21, accuracy of liquid viscosity evaluation was 20% and that for the density was 6%. On the background of the methods mentioned above, our method presented in this work gives good results.

In the case of non-Newtonian liquids (e.g., viscoelastic), Inverse Problem solution algorithm is similar to that in the case of Newtonian liquids. In the first stage, an equation of motion (Direct Problem) for the considered structure (waveguide + viscoelastic liquid) must be solved anew. Subsequently, the Inverse Procedure should be performed. The Inverse Procedure is the same as in the case of a Newtonian liquid. In addition, as a result of the Inverse Procedure, besides the viscosity and density of the liquid, we obtain liquid elasticity. Solution of the Inverse Problem in the case of a viscoelastic liquid will be the subject of future works of the authors.

VII. CONCLUSIONS

In the paper, the Sturm-Liouville direct problem for the Love wave propagating in an elastic layered waveguide loaded on the surface by a viscous liquid was formulated and solved. Subsequently, the inverse problem for the ultrasonic Love wave propagating in the considered waveguide structure was also formulated and solved. The inverse problem was formulated as an optimization problem. For this purpose, the appropriate objective function was constructed. Minima of this objective function were evaluated by using the appropriate optimization procedures (Nelder-Mead, nonlinear simplex). These minima determine simultaneously sought values of the viscosity η and density ρ_l. These values of liquid viscosity and density constitute the solution of the inverse problem.

Objective function includes experimental data (v_j^m and α_j^m) and values of phase velocity and attenuation of Love waves (v_j^c and α_j^c) calculated by solving the direct problem. Calculated values (v_j^c and α_j^c) are functions of unknown viscosity η and density ρ_l of a viscous liquid, respectively. These values (v_j^c and α_j^c) depend also on the material parameters of the waveguide, that are known "a priori." Objective function is a measure of the distance between experimental data and values obtained from the theoretical model. Minimization of this distance allows to specify simultaneously the unknown values of liquid viscosity and density.

The solution of the Inverse Problem enables simultaneous determination with good accuracy (of the order of 5%), both the density and viscosity of a viscous liquid. The accuracy of the obtained results can be improved by averaging, i.e., by solving the Inverse Problem for a series of simulated experimental dispersion curves, and consequently, by calculating the mean value of the obtained values of liquid viscosity η and density ρ_l.

The simultaneous determination of both liquid viscosity and density can be of high relevance in the industrial applications to monitor the parameters of liquids during the course of technological processes.

The results obtained in this paper are original and fundamental and can provide essential data for the design and development of Love-wave based miniaturized liquid sensing devices, such as bio and chemo-sensors. Results of the work can also be employed in geophysics and seismology.

Simultaneous determination of the viscosity and density of liquids is of great importance in the monitoring of technological processes in the chemical, food, pharmaceutical, and polymer industries.

According to the authors' knowledge, formulation and solution of the Inverse Problem of the ultrasonic Love wave propagation in a layered elastic waveguide loaded with a viscous liquid, and consequently, simultaneous evaluation of the viscosity η and density ρ_l of Newtonian liquid is a novelty.

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