Program Digest, 2019 IEEE International Ultrasonics Symposium (IUS)
Glasgow, Scotland, October 6-9, 2019

Search for Optimum Parameters of Love Wave Sensors. Development of Exact Analytical Formulas for Sensor Sensitivities

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***Abstract*—In this work we analyze basic characteristics of Love wave sensors implemented in waveguide structures composed of a lossy viscoelastic surface layer deposited on a lossless elastic substrate. It has to be noted that Love wave sensors working at ultrasonic frequencies have the highest mass density sensitivity $S_\sigma^{v_p}$ among all known ultrasonic sensors, such as QCM, Lamb wave or Rayleigh wave sensors. In this paper we have established an exact analytical formula for the mass density sensitivity $S_\sigma^{v_p}$ of the Love wave sensors in the form of an explicit algebraic expression. Subsequently, using this developed analytical formula, we compared theoretically the mass density sensitivity $S_\sigma^{v_p}$ for various Love wave waveguide structures, such as: (1) lossy PMMA surface layer on lossless Quartz substrate and (2) lossy PMMA on lossless Diamond. The performed analysis shows that the mass density sensitivity $S_\sigma^{v_p}$ (real and imaginary part) for a sensor with a structure PMMA on Diamond is five times higher than that of a PMMA on Quartz structure. It was found that the mass density sensitivity $S_\sigma^{v_p}$ for Love wave sensors increases with the increase of the ratio: bulk shear wave velocity in the substrate to bulk shear wave velocity in the surface layer.**

Keywords—Love wave sensors, mass sensitivity, dispersion equation, viscoelastic layers

I. INTRODUCTION

Ultrasonic waves (bulk and surface), widely used in sensing applications, have been successfully employed in variety of sensors, such as biosensors, chemosensors and other sensors measuring large number of physical quantities, such as humidity, viscosity, etc. [1-6]. Sensors which utilize surface waves of the Love type offer many advantages over the sensors employing other types of ultrasonic waves [7-14]. Primarily, Love wave sensors provide the highest sensitivity to mass loading, comparing to sensors using other acoustic (surface and bulk) waves.

The mass density sensitivity $S_\sigma^{v_p}$ is one of the most important parameters characterizing operation and applicability of the ultrasonic sensor, for measurements in liquid environment. In design of Love wave sensors we seek to find a sensor's configuration, for which the output of the sensor is significantly altered by presence of an extra mass layer loading the upper surface of the sensor's waveguide. In fact, large changes in ultrasonic velocity Δv_p and/or attenuation $\Delta\alpha$ of the Love wave will lead to higher accuracy of measurements with Love wave sensors. In Love wave biosensors, working in a liquid environment, a thin mass layer can be built on the sensor surface due to interactions of the initially deposited recognition layer with an investigated analyte, extracted directly from the surrounding liquid.

In this work, we have performed theoretical analysis and numerical calculations for the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors, operating at ultrasonic frequencies. Love wave sensors show the highest sensitivity to mass load, compared to sensors using other acoustic (surface and bulk) waves.

Mass sensitivity is a fundamental parameter that determines the quality of the ultrasonic Love wave sensors. Up to date, the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors has been determined only approximately employing a number of simplifying assumptions.

In this work, we derive for the first time the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors in the form of an exact analytical formula, without referring to any simplifications and limitations.

Huge advantage of an analytical formula for the mass density sensitivity $S_\sigma^{v_p}$ is its capability to reveal explicitly functional dependencies of the sensitivity $S_\sigma^{v_p}$ as a function of all material and geometrical parameters of the Love wave waveguide, such as moduli of elasticity and density of the surface layer and substrate, thickness of the surface layer, as well as wave frequency. Such an insight cannot be easily achieved with pure numerical methods. Therefore, a necessity for laborious and extensive numerical calculations has been greatly reduced.

II. Geometry of the Love wave waveguide

The layered Love wave waveguide structure, analyzed in this paper (see Fig.1) represents a physical model of the Love wave sensor.

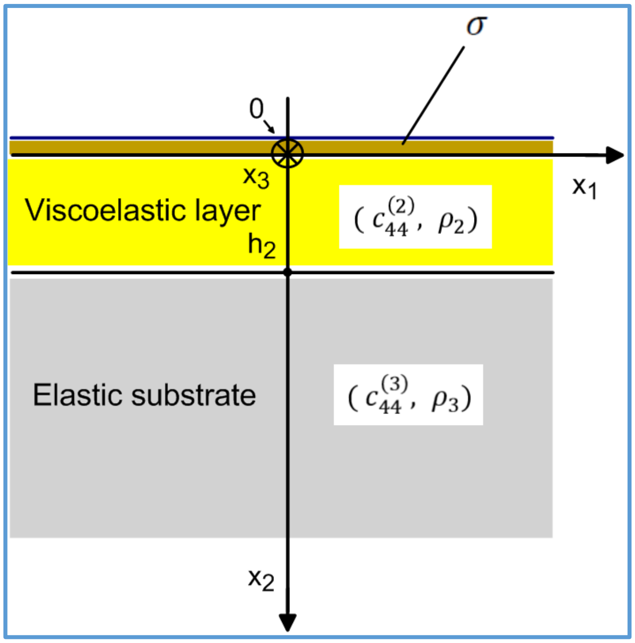

Fig.1. Cross-section of the analyzed Love wave waveguide loaded with a surface mass density σ. Love surface waves propagate along the x_1 axis. Shear horizontal (SH) mechanical displacement u_3 of the Love wave is directed along the x_3 axis.

The waveguide is designed to support shear horizontal (SH) surface waves of the Love type, when the phase velocity of shear ultrasonic waves in the surface layer is lower than that in the substrate. The composite waveguide consists of a lossy viscoelastic surface layer $(h_2 > x_2 \geq 0)$, which is rigidly bonded to a lossless infinite elastic substrate occupying the lower half-space $(x_2 > h_2)$.

The lossy surface layer being a viscoelastic material, such as PMMA (Poly(Methyl Methacrylate)), is characterized by a complex shear modulus of elasticity $c_{44}^{(1)}$. By contrast, the lossless substrate, is a semi-infinite elastic medium, such as ST-cut Quartz, AlN (Aluminum Nitride), BN (Boron Nitride) and Diamond, with real shear modulus of elasticity equal to $c_{44}^{(2)}$. It is well known from previous research that the above materials can support pure SH bulk waves [15, 16] with no spurious components of vibrations along u_1 and u_2 axes.

An important property of Love surface waves is their unique vibration pattern. In fact, Love surface waves have only one non-zero shear-horizontal (SH) component of the mechanical displacement u_3, which is directed along the x_3 axis, parallel to the free surface $(x_2 = 0)$ of the waveguide and perpendicular to the direction of the Love wave propagation along the x_1 axis.

The x_2 axis is directed into the bulk of the substrate. All material parameters of the composite waveguide may change only along the x_2 axis but are homogeneous and isotropic along the x_1 and x_3 axes.

Love wave that propagates in a lossy layered waveguide from Fig.1 undergoes attenuation. Consequently, the wave number k of the Love wave is a complex quantity:

$$k = k_0 + j\alpha \quad (1)$$

where: $j = \sqrt{-1}$ is the imaginary unit, k_0 is the real part of the complex wavenumber which determines the velocity of the Love wave propagation, α is the attenuation of the Love wave and ω is the angular frequency.

III. Mathematical model

The Love wave propagating in the waveguide structure from Fig.1 is governed by the appropriate equations of motion [17, 18] in the constituent regions and the appropriate boundary conditions [19] on the upper and lower surfaces of the surface layer.

Mathematical model of the propagation of Love waves in the waveguide structure from Fig.1 constitutes the following complex dispersion equation:

$$tan(q_2 \cdot h_2) \cdot \left\{ \left(c_{44}^{(2)} \cdot q_2\right)^2 + (\sigma \cdot \omega^2) \cdot \left(c_{44}^{(3)} \cdot b\right) \right\} + \\ + \left(c_{44}^{(2)} \cdot q_2\right) \cdot \left\{ (\sigma \cdot \omega^2) - \left(c_{44}^{(3)} \cdot b\right) \right\} = 0 \quad (2)$$

where: $k_2 = \frac{\omega}{v_2}$; $k_3 = \frac{\omega}{v_3}$; $q_2 = \sqrt{k_2^2 - k^2}$; $b = \sqrt{k^2 - k_3^2}$; σ is the surface mass density loading the surface of the waveguide and k is the complex wavenumber of the Love wave.

Equation No. 2 relates the phase velocity and attenuation of the Love wave propagating in the waveguide with material and geometrical parameters of the layered waveguide, depicted in Fig.1. It should be stressed that an extra mass loading does not introduce any extra losses, but only changes in the phase velocity of the Love wave.

IV. Mass sensitivity $S_\sigma^{v_p}$

Equation (2) can be regarded as an implicit function of the phase velocity v_p, attenuation α surface mass density σ and other parameter of the waveguide. The sensitivity $S_\sigma^{v_p}$ of the Love wave sensor to the surface density mass loading can be defined as [3]:

$$S_\sigma^{v_p} = \frac{1}{v_p}\left(\frac{\partial v_p}{\partial \sigma}\right) \quad (3)$$

For lossy waveguide structures, the phase velocity of the Love wave can be defined as a complex quantity: $v_p = \omega/k$. As a result, the mass density sensitivity defined by Eq. (3) becomes also a complex quantity.

Employing Eq.2 and the theorem of differentiation of implicit functions, we have derived for the first time the following explicit formula for the mass density sensitivity $S_\sigma^{v_p}$:

$$S_\sigma^{v_p} = \frac{\omega^2 \frac{1}{k}\left\{\left(c_{44}^{(2)} q_2\right)+\left(c_{44}^{(3)} b\right)\cdot tan(q_2 h_2)\right\}}{\frac{h_2}{cos^2(q_2 h_2)}\frac{\partial q_2}{\partial k}\left\{\left(c_{44}^{(2)} q_2\right)^2+\left(c_{44}^{(3)} b\right)(\sigma\omega^2)\right\}+tan(q_2 h_2)\left\{2q_2\left(c_{44}^{(2)}\right)^2\frac{\partial q_2}{\partial k}+c_{44}^{(3)}\frac{\partial b}{\partial k}(\sigma\omega^2)\right\}+c_{44}^{(2)}\frac{\partial q_2}{\partial k}\cdot\left\{(\sigma\omega^2)-\left(c_{44}^{(3)} b\right)\right\}-c_{44}^{(3)}\frac{\partial b}{\partial k}\left(c_{44}^{(2)} q_2\right)} \quad (4)$$

where: $\frac{\partial q_2}{\partial k} = -\frac{k}{\sqrt{k_2^2-k^2}}$; $\frac{\partial b}{\partial k} = \frac{k}{\sqrt{k^2-k_3^2}}$

Equation (4) is an exact analytical formula for the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors.

Equation (4) will be further used in the subsequent numerical calculations of the mass density sensitivity $S_\sigma^{v_p}$ of the Love wave sensors.

V. RESULTS

Using the derived analytical formula (Eq.4) we have calculated numerically the real part of the mass density sensitivity $S_\sigma^{v_p}$ of the Love wave sensor, for various combinations of the surface layer and substrate materials. As a substrate material, ST-cut Quartz and Diamond were chosen.

The real part of the mass density sensitivity $S_\sigma^{v_p} = \frac{1}{v_p}\left(\frac{\partial v_p}{\partial \sigma}\right)$, as a function of frequency f of the Love surface wave and thickness h_2 of PMMA guiding surface layer is given, respectively, in Figs 2 and 3. Here, as a substrate we chose an ST-cut Quartz material.

It is apparent that the real part of the mass density sensitivity $S_\sigma^{v_p}$ displays resonant like peaks, as a function of both f and h_2, and drops gradually to zero for higher frequencies, if $f \to +\infty$.

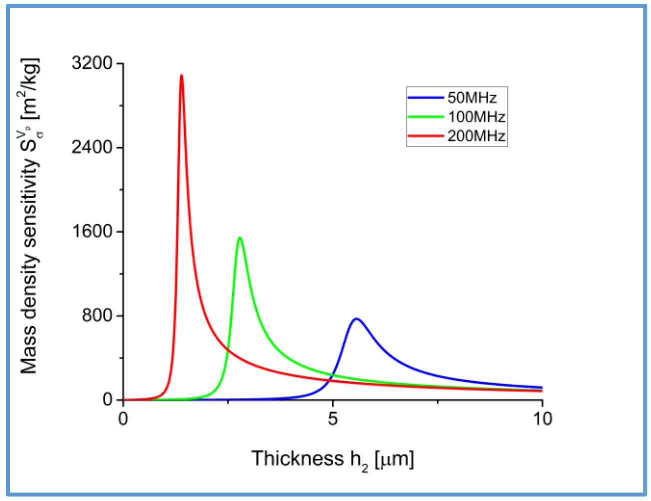

Fig.3. Real part of the mass density sensitivity $S_\sigma^{v_p}$ $[m^2/kg]$ for Love surface waves propagating in PMMA-ST-Quartz waveguides, loaded with a thin lossless film with the mass density σ, as a function of thickness h_2 of the PMMA guiding surface layer, for different values of wave frequency $f = 50, 100$ *and* 200 MHz.

Figures 4 and 5 show dependencies of the real part of the mass density sensitivity $S_\sigma^{v_p}$ as a function of frequency f and thickness h_2 for the PMMA surface layer deposited this time on a diamond substrate.

Love wave sensors based on PMMA on Diamond structures have a significantly larger mass sensitivity (five times) than sensors using PMMA on Quartz structures.

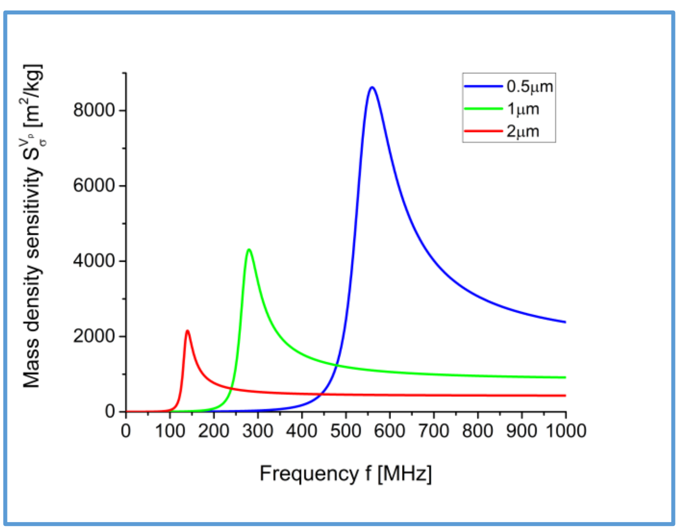

Fig.2. Real part of the mass density sensitivity $S_\sigma^{v_p}$ $[m^2/kg]$ for Love surface waves propagating in PMMA-ST - Quartz waveguides loaded with a thin lossless film with the surface mass density σ, as a function of Love wave frequency f, for different values of thickness h_2 of the guiding PMMA surface layer ($h_2 = 0.5, 1$ and 2 μm).

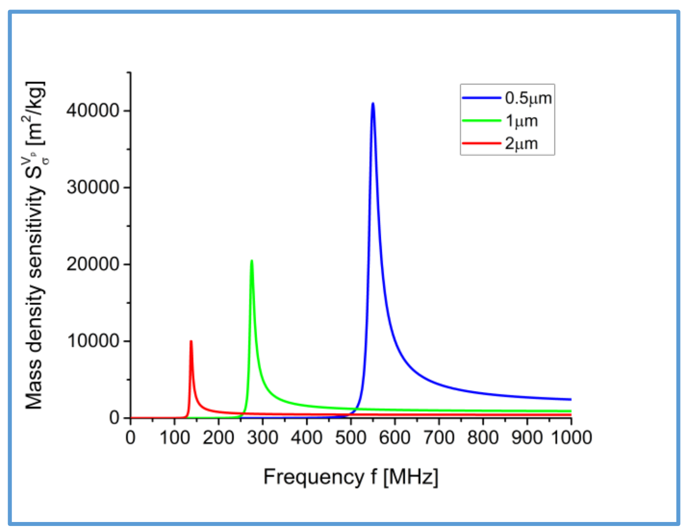

Fig.4. Real part of the mass density sensitivity $S_\sigma^{v_p}$ $[m^2/kg]$ for Love surface waves propagating in PMMA - Diamond waveguides loaded with a thin lossless film with the mass surface density σ, as a function of Love wave frequency f, for different values of thickness h_2 of the guiding PMMA surface layer ($h_2 = 0.5, 1$ *and* 2 μm).

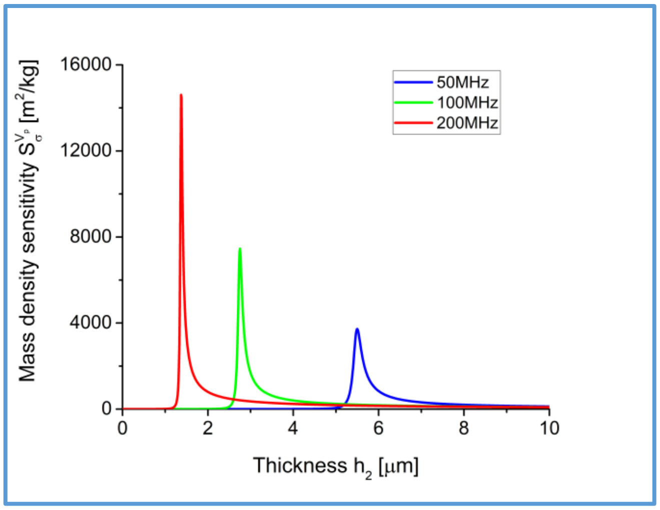

Fig.5. Real part of the mass density sensitivity $S_\sigma^{v_p}$ $[\mathrm{m}^2/\mathrm{kg}]$ for Love surface waves propagating in PMMA-Diamond waveguides, loaded with a thin lossless film with the surface mass density σ, as a function of thickness h_2 of the PMMA guiding surface layer, for different values of wave frequency $f = 50, 100\ and\ 200\ MHz$.

VI. Conclusions

From the theoretical analysis and numerical calculations performed in this work, we can draw the following conclusions:

1) the real part of the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors can be optimized (maximized) by proper selection of material and geometric parameters of the layered waveguide structure

2) for a constant frequency f, the real part of the mass density sensitivity $S_\sigma^{v_p}$ reaches a maximum as a function of thickness h_2 of the surface layer

3) for a constant thickness h_2 of the surface layer, the real part of the mass density sensitivity attains a maximum as a function of frequency f

4) the real part of the mass density sensitivity $S_\sigma^{v_p}$ of the Love wave sensors increases with growing velocity ratio v_2/v_1, where: v_1 is the bulk shear wave velocity in the surface layer, and v_2 is the bulk shear wave velocity in the substrate

5) a more complete optimization process of the mass density sensitivity $S_\sigma^{v_p}$ of Love wave sensors requires further theoretical and numerical investigations.

Acknowledgment

The project was funded by the National Science Centre (Poland), granted on the basis of Decision No. 2016/21/B/ST8/02437.

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