The influence of rheological parameters of viscoelastic liquids on the propagation characteristics of ultrasonic Love waves

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Abstract— **Progress in materials science has led to development of new materials with improved functional characteristics. One of the new types of materials introduced into industrial practice are plastics and polymers. These materials exhibit rheological (viscoelastic) properties, which combine simultaneously the properties of liquids and solids. Due to their attractive features, such as low specific weight, high resistance to chemical agents, ease of processing, cost effectiveness etc. these materials are widely used in chemical, automotive, aviation and space industry. In addition, these materials are very common in many aspects of everyday life. Thus, it is very important, both from the theoretical and practical point of view, to develop new, robust and accurate methods to measure the rheological parameters (viscosity η, elasticity μ and density ρ) of plastics and polymers. The conventional mechanical methods used so far to this end are outdated, time consuming, and cumbersome. Ultrasonic methods do not possess these disadvantages. The first step in the formulation of the Inverse Method for evaluating the rheological parameters of viscoelastic liquids is to formulate and solve the Direct Sturm-Liouville Problem for Love waves propagating in the investigated layered elastic waveguide loaded on its surface with various types of viscoelastic materials (e.g., liquids). The aim of this study is to develop a rigorous mathematical model (Direct Sturm-Liouville Problem) of propagation of shear horizontal (SH) surface Love waves in layered viscoelastic structures, i.e., in layered elastic waveguides with a guiding surface layer covered with a viscoelastic material described by Kelvin-Voigt, Newton and Maxwell viscoelastic models respectively.**

Keywords—Love waves; viscoelastic liquid; Sturm-Liouville problem; dispersion curves

I. INTRODUCTION

The development of materials science has led to creation of new materials with improved functional properties. One of the new types of materials introduced into industrial practice are plastics and polymers. These materials exhibit the viscoelastic rheological properties, i.e., they have simultaneously properties of liquids and solids.

It is very important both from a cognitive and practical point of view, to develop new and accurate methods of measuring the rheological parameters (viscosity η, elasticity μ and density ρ) of plastic and polymers. New materials require new methods of measuring their rheological parameters.

For the measurement of rheological parameters of viscoelastic media so far mechanical methods are used. These methods are very cumbersome, to some extent outdated, time consuming and destructive.

To this end, we propose in this paper to employ surface acoustic waves of the Love type, for an on-line investigation of rheological parameters of viscoelastic media. Ultrasonic waves, e.g., that of the Love type are widely used in sensor technology [1-6] and NDT of materials [7-12]. The main advantages of the Love wave method compared to traditional mechanical methods of material testing are the following: 1) it is non-destructive, 2) it is fast and accurate, 3) it can be computerized and 4) there are no moving parts.

Shear horizontal (SH) surface waves of the Love type can propagate in surface waveguides composed of a surface layer of finite thickness "D" rigidly bonded to a semi-infinite substrate. If the surface layer and substrate is lossless, Love waves propagate without attenuation. However, if the surface layer is loaded with a lossy viscoelastic material the attenuation of Love waves will be non-zero, since part of their energy will be gradually transferred from the waveguide to the loading material. The coefficient of attenuation α is a function of wave frequency, surface layer thickness "D", and the rheological parameters of the loading viscoelastic liquid (material).

Lossy viscoelastic materials can be described by a number of rheological models, such as Kelvin-Voigt, Newton and Maxwell model. In this study we analyze how different rheological models of a lossy loading material affect an overall attenuation of Love waves propagating in lossless, elastic, layered waveguides loaded with lossy materials of different types.

The first step in the formulation of the Inverse Method for the determination of rheological parameters of viscoelastic liquids is the formulation and solution of the Direct Sturm-Liouville Problem for Love waves propagating in the investigated waveguide structure, see Fig.1. Solution of the Direct Sturm-Liouville Problem consists in determining the Love wave dispersion curves, (i.e., the dependence of the phase velocity and attenuation on frequency) for given material and

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geometric parameters of the layered waveguide and viscoelastic liquid.

The objective of this work is to establish a mathematical model of propagation of Love waves in layered elastic waveguides covered on their surface with viscoelastic materials described by different viscoelastic models, such as Kelvin-Voigt, Newton and Maxwell models. To this end, we developed a corresponding complex dispersion equation for Love waves propagating in loaded waveguides and performed adequate numerical calculations.

The results of this study should be useful for designers and scientists working in geophysics, microelectronics (MEMS [13], biosensors [14], chemosensors [15]), mechanics of materials and biomechanics.

II. MATHEMATICAL FORMULAS

Dispersion curves of the Love wave, i.e., the dependence of the phase velocity and attenuation on frequency, result from solutions of the corresponding boundary value problem, called the direct Sturm-Liouville problem [16]. In fact, the dispersion curves are functions of the wave frequency, material parameters of the constituent elements of the waveguide (substrate, surface layer and loading material) as well as of the thickness of the suface layer itself.

Love waves can only propagate in layered structures, e.g., in elastic waveguides composed of an elastic surface layer rigidly bonded to an elastic substrate. The top of the surface layer $x_2 = -D$ is loaded with a viscoelastic liquid (medium), see Fig.1.

Love surface waves, propagating in isotropic waveguides, have only one component of vibrations u_3, polarized along the axis x_3 that is perpendicular to the direction of propagation x_1.

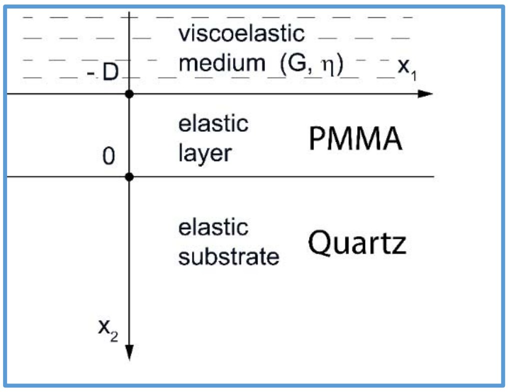

Fig.1. Lossless (elastic) Love wave waveguide (surface layer plus substrate) loaded at the surface $x_2 = -D$ with a lossy viscoelastic medium of the shear modulus G and viscosity η.

III. RHEOLOGICAL MODELS OF VISCOELASTIC MEDIA

In this work, we have chosen three different models for the viscoelastic media (i.e., Newton, Kelvin-Voigt, and Maxwell models) [17], which load the surface of the Love wave waveguide.

For time-harmonic waves rheological properties of viscoelastic media can be represented by a complex shear modulus c_{44}^L.

Employing constitutive equations for the three viscoelastic liquids (media) considered, we arrive at the following three formulas for the complex shear moduli of elasticity:

1) Newton model:

$$c_{44}^L = -j\omega\eta \quad (1)$$

where: η is the viscosity of the viscoelastic medium,

2) Kelvin-Voigt model:

$$c_{44}^L = G - j\omega\eta = G(1 - jtan\delta) \quad (2)$$

where: G is the elastic shear modulus, and η is the viscosity of a viscoelastic medium, $tan\delta = \omega\eta/G$

3) Maxwell model:

$$c_{44}^L = G\frac{(\omega\tau)^2}{1+(\omega\tau)^2} - jG\frac{\omega\tau}{1+(\omega\tau)^2} \quad (3)$$

where: $\tau = \eta/G$ is the relaxation time of the Maxwell medium.

Equations 1-3 show that the properties of the Maxwellian material (liquid-like) are to some extent inverse to those of the Kelvin-Voigt material (solid-like). Namely:

a) in the low frequency limit $tan\delta \ll 1$ the Kelvin-Voigt material behaves like a solid material with the modulus of elasticity G and the Maxwellian material behaves like a Newtonian liquid with the viscosity η

b) in the high frequency limit $tan\delta \gg 1$ the Kelvin-Voigt material approaches a Newtonian liquid with the viscosity η and the Maxwellian material is a solid like with the modulus of elasticity G.

IV. COMPLEX DISPERSION EQUATION FOR LOVE WAVES

The complex dispersion equation (Eq.4) for Love waves that propagate in the considered waveguide structure depicted in Fig.1 is obtained by substitution of the developed propagation Love wave solutions (in the substrate, layer and viscoelastic medium respectively) into the boundary conditions and finally by equating to zero the resultant determinant. As a result, it can be shown that the following complex dispersion equation holds:

$$\sin(qD)\cdot\{(\mu_1)^2\cdot q^2 - \mu_2\cdot b\cdot\lambda_1\cdot c_{44}^L\} - \cos(qD)\cdot\mu_1\cdot q\cdot\{\mu_2\cdot b + \lambda_1\cdot c_{44}^L\} = 0 \quad (4)$$

where: complex pairs of quantities $\mu_{2,}\, b,\ \mu_{1,}\, q, and\ \lambda_1, c_{44}^L$ correspond to the substrate, surface layer and the loading material, respectively, and $j = (-1)^{1/2}$.

The above equation is a nonlinear transcendental algebraic equation for the complex propagation constant $k = k_0 + j\alpha$ as unknown.

Solving equation 4 (the solution is a pair $(k_0,\ \alpha)$) allows for determination of the phase velocity of the Love wave $v = \omega/k_0$

and the imaginary part α of the complex wavenumber k representing the attenuation in Np/m of the Love wave per unit length in the direction of propagation x_1.

V. Dispersion Curves of Love Waves

Numerical calculations were performed for the waveguide structure shown in Fig.1 with the parameters given below. The frequency of the Love wave varied from 1 to 1000 MHz, what is equivalent to $tan\delta = \omega\eta/G$, ranging from 0.127 to 127.

In the numerical calculations the following values of material parameters were employed:

For PMMA poly(methyl methacrylate)

$\mu_1 = 1.43 \times 10^9 \ N/m^2$; $\rho_1 = 1.18 \times 10^3 \ kg/m^3$

$v_1 = (\mu_1/\rho_1)^{1/2} = 1100 \ m/s$

For Quartz (ST-cut 90º X)

$\mu_2 = 67.85 \times 10^9 \ N/m^2$; $\rho_2 = 2.56 \times 10^3 \ kg/m^3$

$v_2 = (\mu_2/\rho_2)^{1/2} = 5060 \ m/s$

Thickness D of the surface layer equaled 0.1 mm. The density of the considered viscoelastic materials was assumed as $\rho_l = 1 \times 10^3 \ kg/m^3$.

Losses in the PMMA layer and Quartz substrate were neglected. The only source of losses is the viscosity of the viscoelastic medium that charges the surface of the waveguide.

A. Attenuation Curves

Figure 2 shows the dependence of the attenuation of Love surface waves on frequency for three types of media (i.e., Kelvin-Voigt, Newton and Maxwell) that load the waveguide. The frequency of the wave is in the low frequency limit 0-10 MHz, $tan\delta < 1$.

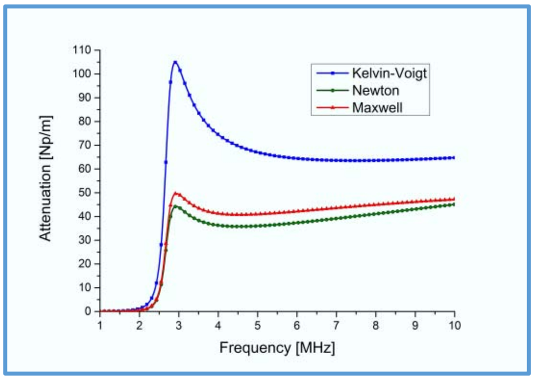

Fig. 2 Attenuation of the Love surface wave, propagating in a lossless elastic waveguide loaded with three different types of lossy viscoelastic materials, i.e., Kelvin-Voigt, Newton and Maxwell, in the low frequency limit: $tan\delta \in [0.127 - 1.27]$, $G = 5 \cdot 10^4 Pa, \ \eta = 1 \, mPas$.

Figure 3 presents the attenuation of Love waves, in the range of low and medium frequencies; from 0 to 100 MHz.

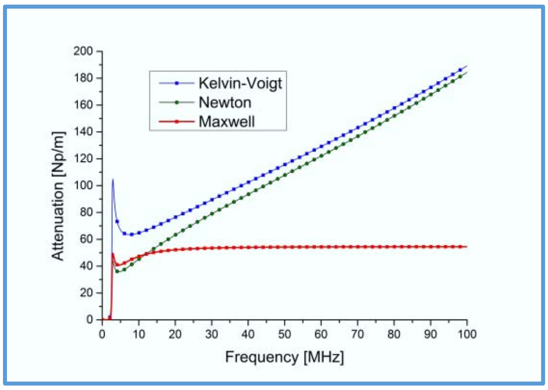

Fig.3. Attenuation of the Love surface wave, propagating in a lossless elastic waveguide, loaded with 3 different types of lossy viscoelastic materials, i.e., Kelvin-Voigt, Newton and Maxwell. Low and medium frequency limits: $tan\delta \in [0.127 - 12.7]$, $G = 5 \cdot 10^4 Pa, \ \eta = 1 \, mPas$.

B. Phase Velocity Curves

Figure 4 exhibits a graph of the phase velocity v_p of the Love wave, versus frequency. Since the phase velocity v_p is almost independent of the type of the loading material considered, the individual phase velocity plots for Kelvin-Voigt, Newton and Maxwell materials are indistinguishable in Fig. 4.

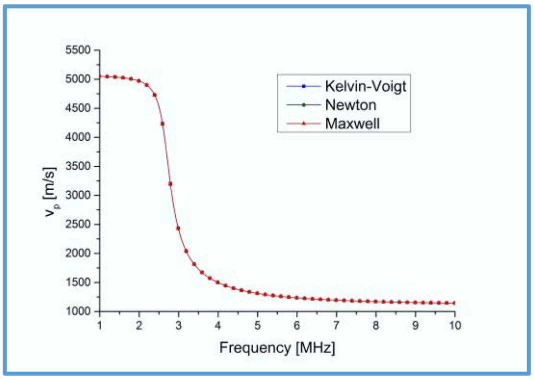

Fig. 4 Phase velocity of the Love surface wave, propagating in a lossless elastic waveguide loaded with three different types of lossy viscoelastic materials, i.e., Kelvin-Voigt, Newton and Maxwell, in low frequency limit: $tan\delta \in [0.127 - 1.27]$, $G = 5 \cdot 10^4 Pa, \ \eta = 1 \, mPas$.

VI. Discussion

From Eqs. 1 and 3 it is clear that in the low frequency range when $tan\delta = \omega\eta/G$, changes from 0.127 to 1.27, the properties of the Maxwellian liquid approaches those of the Newtonian liquid. Correspondingly, the attenuation of the Love wave due to the load with Maxwellian and Newtonian liquids is almost the same (see Fig. 2). By contrast, the attenuation corresponding to the Kelvin-Voigt material is about two times higher, in the same frequency range.

In the medium frequency range for $tan\,\delta = \omega\eta/G$, changing from 1.27 to 12.7, the attenuation due to the Newtonian liquid starts to rise quickly and for frequencies higher than 25 MHz is closer to that corresponding to the Kelvin-Voigt material (see Fig. 3). By contrast, the attenuation due to Maxwellian liquid reaches a maximum and starts to decrease since for growing frequencies the Maxwellian liquid approaches a solid like elastic body without viscosity.

In the high frequency range for $tan\,\delta = \omega\eta/G$, changing from 12.7 to 127, the attenuation due to the Maxwellian liquid decreases continuously and is a few orders of magnitude lower than that corresponding to the Newtonian liquid and Kelvin-Voigt material (see Fig. 3), which grows quadratically with frequency of the wave, as expected.

It is noteworthy that the phase velocity of the Love wave weakly depends on the type of the loading material and its viscoelastic parameters, such as the storage modulus G and viscosity η (see Fig. 4).

VII. Conclusions

The attenuation of the Love surface wave propagating in a lossless elastic waveguide, loaded with various types of viscoelastic materials depends significantly on the type of the loading medium, described either by Kelvin-Voigt, Newton or Maxwell viscoelastic models, and on the wave frequency.

The theoretical analysis and the results of numerical calculations presented in this paper reveal that the attenuation of the Love wave reflects directly the viscoelastic properties of the loading material described by Kelvin-Voigt, Newton and Maxwell models. Namely:

a) in the low frequency limit $tan\delta \ll 1$ the attenuation of the Love wave due to the Maxwellian liquid and that due to the Newtonian liquid are almost the same (see notice "a" in Section III and Fig.2)

b) in the high frequency limit $tan\delta \gg 1$ the attenuation of the Love wave due to the Kelvin-Voigt material and that due to the Newtonian liquid are almost identical (see notice "b" in Section III and Fig.3).

The results obtained in this work are of crucial importance in assessment of measurement data obtained with Love wave sensors in wide range of frequencies and viscosities measured. Moreover, these results can provide essential data for the design and development of the SH Love wave based bio and chemosensors.

It is noteworthy that propagation of Love surface waves in layered elastic waveguides loaded with viscoelastic media (materials) is also of paramount importance in seismology [18] and geophysics [19].

To our best knowledge, evaluations of dispersion curves for Love waves propagating in waveguides loaded with viscoelastic materials of different types (Kelvin-Voigt, Newton and Maxwell) is an original contribution of the Authors to the state-of-the-art.

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