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Inverse problem of the Love wave propagation in elastic waveguides loaded with a viscous liquid

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Abstract – **The problem of propagation of Love waves in elastic waveguides loaded on the surface by a viscous (Newtonian) liquid is important in many applications such as geophysics, seismology, investigation of the physical properties of liquids. Love wave energy is concentrated near the waveguide surface, so that Love waves are especially suited to study the material properties of surface layers. In this work, the direct problem and the inverse problem of the Love wave propagation in a layered elastic waveguides loaded with a viscous liquid have been formulated and solved. The inverse problem relies on the determination of the material parameters (e.g., the unknown value of liquid viscosity) from measurements of the dispersion curves of Love waves.**

Keywords: Love wave, inverse problem, viscosity, elastic waveguide

I. INTRODUCTION

The problem of propagation of Love waves in elastic waveguides loaded on the surface by a viscous (Newtonian) liquid is important in many applications such as geophysics [1], seismology [2], earthquake engineering [3], sensors [4], and in the NDT of materials [5,6]. Love wave energy is concentrated near the waveguide surface, so that Love waves are especially suited to study the material properties of surface layers. In this work, the direct problem and the inverse problem of the Love wave propagation in a layered elastic waveguide loaded with a viscous liquid have been formulated and solved. The solution of the inverse problem allows to determine the unknown value of liquid viscosity from measurements of the dispersion curves of Love waves.

The results obtained in this paper are fundamental and can provide essential data for the design and development of Love-wave-based liquid sensing devices.

II. DIRECT STURM-LIOUVILLE PROBLEM (LOVE WAVES)

Calculation of the dispersion curves and amplitude of a surface wave for given values of liquid viscosity and elastic parameters of the surface layer and substrate forms a direct Sturm-Liouville problem. Love wave propagates in a layered elastic waveguide (an elastic isotropic layer is rigidly attached to an isotropic and elastic substrate). The top surface of the layer is loaded with a viscous (Newtonian) liquid, see Fig.1.
The mechanical displacement of the Love wave u_3 is polarized along the x_3 axis, perpendicular to the direction of propagation x_1. The waveguide surface is at $x_2 = -D$.

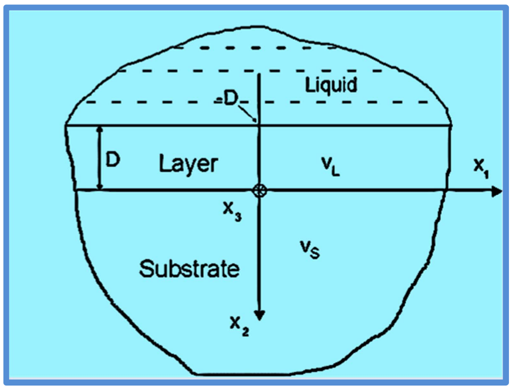

Fig.1. Geometry of a Love wave waveguide ($v_L < v_S$) covered by a viscous liquid.

A. COMPLEX DISPERSION EQUATION

Using the continuity conditions of the mechanical displacement and shear stress, at the interfaces $x_2 = 0$, and $x_2 = -D$, we arrive at the following analytical expression for the complex dispersion equation of the Love wave [6]:

$$\sin(qD) \cdot \{(\mu_1)^2 \cdot q^2 + \mu_2 \cdot b \cdot \lambda_1 \cdot j\omega\eta\} + - \cos(qD) \cdot \{\mu_1 \cdot \mu_2 \cdot b \cdot q - \mu_1 \cdot q \cdot \lambda_1 \cdot j\omega\eta\} = 0 \quad (1)$$

where: quantities q, b and λ_1 are complex. Complex wave number $k = k_0 + j\alpha$, $j = (-1)^{1/2}$.

Equation (21) is complex dispersion equation of Love waves propagating in an elastic layered waveguide loaded on the surface with a viscous Newtonian liquid. The real part of the wave number k_0 determines the phase velocity of the Love wave. The imaginary part of the wave number α is an attenuation coefficient of the Love wave.

After separating the real and imaginary parts of Eq.1, we obtain the following system of nonlinear algebraic equations with unknowns: k_0 and α.

$$A(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega; k_0, \alpha) = 0 \quad (2)$$

$$B(\mu_1, \rho_1, \mu_2, \rho_2, \eta, \rho_l, D, \omega; k_0, \alpha) = 0 \quad (3)$$

Here: μ1, ρ1, μ2, ρ2, η, ρl, D and ω are parameters. The unknowns are: k_0 and α.

After finding a solution (k_0, α), one can calculate the phase velocity of Love wave $v = \omega/k_0$.

II. INVERSE PROBLEM

The inverse problem relies on the determination of unknown material parameters from the measured dispersion curves of shear horizontal surface waves (i.e., Love waves) propagating in the considered layered structure.

To solve the inverse problem the following three steps were performed:

1) formulation and solution of the direct problem

2) numerical experiment (synthetic data)

3) formulation and solution of the inverse problem

In this study, the inverse problem was formulated and solved as an optimization problem [7] with properly defined objective function.

A. OBJECTIVE FUNCTION

The objective function depending on the liquid viscosity, material parameters of the structure, frequency, and experimental data (phase velocity and attenuation of the surface Love wave) was introduced and defined as:

$$\Pi(\eta) = \sum_{j=1}^{N_e} \{ A^2(par, \omega_j, k_0^j, \alpha_j, \eta) + B^2(par, \omega_j, k_0^j, \alpha_j, \eta) \} \quad (4)$$

where: N_e is the number of experimental points, “par" denotes known „a priori” parameters, ω_j is the measured angular frequency, k_0^j is the measured real part of the wave number, α_j is the measured attenuation, η is an unknown liquid viscosity.

Making use of the optimization methods a minimum of the objective function was evaluated. It enabled the determination of the optimum values for the liquid viscosity. These optimum values of the viscosity of the liquid constitute the solution of the inverse problem.

To minimize the considered objective function the quasi-Newton optimization procedure from Mathcad® package was employed.

IV. NUMERICAL CALCULATIONS

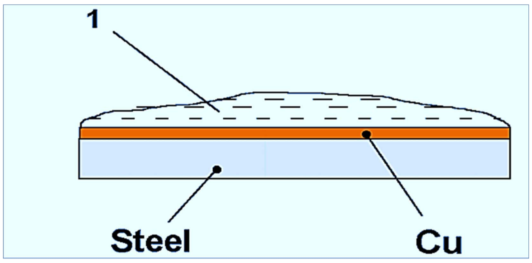

Fig.2. Waveguide (Cu on steel) of the Love wave. Shear surface wave propagates along the waveguide surface. The surface of Cu layer is loaded by a viscous (Newtonian) liquid (1). D = 0.4 mm.

Numerical calculations were performed for the structure: Cu (copper) surface layer (on steel substrate) loaded on its surface with a viscous (Newtonian) liquid, see Fig.2. Losses in the Cu layer and steel substrate were neglected. The only source of losses is the viscosity of the liquid.

V. NUMERICAL EXPERIMENT (SYNTHETIC DATA)

In our numerical experiments we calculated the dispersion curves of the phase velocity and attenuation for a value of liquid viscosity $\eta = 1\,Pas$. Subsequently, we added random errors to the values of phase velocity and attenuation. We considered two values of random errors, i.e.; 0.1% and 0.5%.

VI. RESULTS

In order to solve the Inverse Problem we employed the inverse algorithm described in

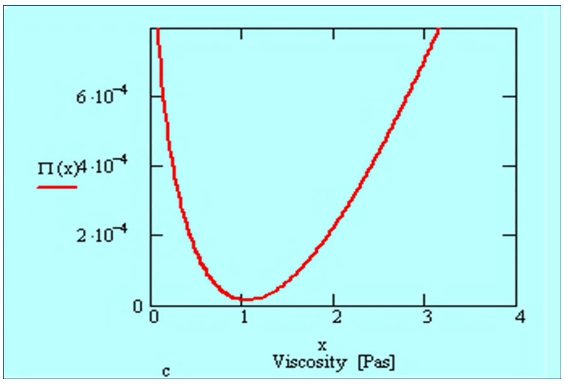

Fig.3. Objective Function $\Pi(\eta)$ as a function of liquid viscosity for maximum random error 0.5%.
Exact value of liquid viscosity $\eta = 1\,Pas$.

Section III. To construct the Objective Function $\Pi(\eta)$ we used the analytical formulas for the real (Eq.2) and imaginary (Eq.3) parts of the complex dispersion equation (Eq.1).

We apply the optimization procedures to determine the minimum (see Fig.3) of the Objective Function $\Pi(\eta)$.

Dispersion curves of phase velocity and attenuation were evaluated for 6 values of frequency:
f = 0.5 MHz, *1.0* MHz, *1.5* MHz, *2.0* MHz, *2.5* MHz *and 3.0* MHz and for liquid viscosity
$\eta = 1\ Pas$.
(Direct Problem)
Minimization of the Objective Function $\Pi(\eta)$ provides the unknown value of liquid viscosity.
(Inverse Problem)

The results of the performed inverse procedure are inserted in Tables I and II.

Calculation No	1	2	3	4	5	6	7	8	average	error
Viscosity [Pa s]	0.822	0.957	0.815	1.035	1.19	1.035	1.242	0.992	1.12	12%

Table I. Viscosity determined from the Inverse Method for the maximum random error = 0.5%.

Calculation No	1	2	3	4	5	6	7	8	average	error
Viscosity [Pa s]	0.911	1.044	1.018	0.929	0.993	1.023	1.075	1.028	1.045	4.5%

Table II. Viscosity determined from the Inverse Method for the maximum random error = 0.1%.

As we can see from Tables I and II, the error of the determination of an unknown value of liquid viscosity depends on the error of performed numerical experiments (Love wave phase velocity and attenuation). For a random error 0.5%, the error of resultant liquid viscosity equal 12%, and similarly for an experimental error equal 0.1%, unknown liquid viscosity was evaluated with error equal 4.5%.

To increase the accuracy of the obtained from the inverse method liquid viscosity, we propose to employ another than that given by Eq.4 types of the objective function.

The calculations of the inverse method using these another types of the objective function will be performed in future works.

VII. CONCLUSIONS

In this work we perform a rigorous mathematical analysis of the problem of the Love wave propagation in elastic layered waveguides covered with a viscous liquid. Effect of liquid viscosity on the phase velocity of Love waves and attenuation is presented and analyzed. Based on the results of the direct problem and experimental data, the inverse problem was formulated as an optimization problem and solved.

In this work:

1) A novel inverse method, employing Love surface waves for the determination of liquid viscosity from measured dispersion curves, was developed.
2) The Direct Problem for Love wave propagation was formulated and solved numerically. Analytical complex dispersion equation has been established.
3) The Inverse Problem was formulated as an optimization problem. Consequently, the Objective Function based on the dispersion equation and experimental data was determined and minimized.
4) The results of investigations demonstrate that the viscosity of liquid can be successfully determined from measurements of Love wave dispersion curves .

The results obtained in this study are fundamental and can be useful in NDT of materials, in geophysics, seismology, in earthquake engineering, and in sensors and biosensors. According to the authors' knowledge, formulation and solution to both the direct and inverse problems of the Love wave propagation in a layered elastic waveguide loaded with a viscous (Newtonian) liquid is a novelty.

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