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Inverse determination of thickness and elastic properties of thin layers and graded materials using generalized Love waves

[a]P. KIEŁCZYŃSKI, M. SZALEWSKI
Institute of Fundamental Technological Research, Polish Academy of Sciences,
ul. Pawińskiego 5B, 02-106 Warsaw, Poland.

[a]e-mail: pkielczy@ippt.gov.pl

Abstract - **Determination of the mechanical and geometrical parameters of thin coatings and surface layers in materials is of great practical importance in engineering and technology. In this work the authors present a novel inversion procedure for simultaneous determination of thickness, shear elastic constant and density of thin coating layers in materials. The inversion procedure is based on measurements of the dispersion curve for Love surface acoustic waves. The inverse problem is formulated as an optimization problem with the appropriately designed objective function, depending on the material parameters of the coating layer, ultrasonic frequency, and the experimental data, i.e., measured phase velocity of the surface Love wave. The minimization of the objective function provides three parameters of a thin layer, i.e., its thickness, shear elastic constant and density. The agreement between the results of calculations with the proposed inversion method and the experimental data was good.**

Keywords: *Love waves; elastic waves, inverse methods, dispersion curves, thin layers*

I. INTRODUCTION

The mechanical properties of coatings deposited on a substrate and surface layers in graded materials are of crucial importance in the design and evaluation in modern engineering practice. Traditional mechanical methods for characterization of the surface properties of materials are tedious, time consuming and what is the most important they are destructive. Employment of bulk and surface acoustic waves provided truly non-destructive tools in material characterization. An important property of all surface waves is the fact that their amplitude decays practically to zero in a few wavelength from the guiding surface. Thus, by changing wave frequency one can probe subsurface profiles of the material. Till present, thin layers in materials were investigated using Rayleigh surface waves [1,2], which possess two perpendicular components of the mechanical displacement. By contrast, Love surface waves have only one shear horizontal (SH) component of vibration and closed-form analytical solutions for Love surface waves exist. Due to their simplicity Love surface waves are very attractive for inverse problem applications, where one must calculate many times the corresponding direct problem solution. Love surface waves are used in geophysics to study mechanical properties of selected geological structures [3]. In this work, the theoretical and experimental investigations have been carried out on the following layered structure: thin copper (Cu) layer deposited electrolytically on a steel substrate.

The inverse problem constitutes determination of unknown material parameters from the measured dispersion curves (phase velocity as a function of frequency) for Love waves. To solve the inverse problem one has to perform the following steps: 1) solve direct problem, 2) determine experimentally dispersion curves, 3) perform the inverse procedure.

Calculation of Love wave parameters (e.g., phase velocity, distribution of the wave amplitude with depth) for known a priori values of material parameters of the layer and substrate constitutes the direct problem. In this study, the direct problem (Sturm-Liouville problem) was formulated and solved.

In the present work the inverse problem was formulated and solved as an optimization problem. The objective function depending on the material parameters of the structure, frequency, and experimental data (dispersion curves of the surface wave) was developed. The dispersion curves were measured in the computerized measuring set-up. Making use of the optimization methods a minimum of the objective function was determined. This enabled the determination of the unknown mechanical parameters such as shear elastic coefficients and thickness of thin coating films. The obtained from the inverse method elastic and geometrical parameters of thin films were used as input data in the calculations of the direct problem. Resulting from the direct problem dispersion curves were compared with those measured experimentally. Good conformity between theoretical and experimental dispersion curves has been stated.

II. DIRECT STURM-LIOUVILLE PROBLEM

Calculation of the dispersion curves and amplitude of a surface wave for given values of elastic parameters of the surface layer and substrate forms a direct problem. The direct

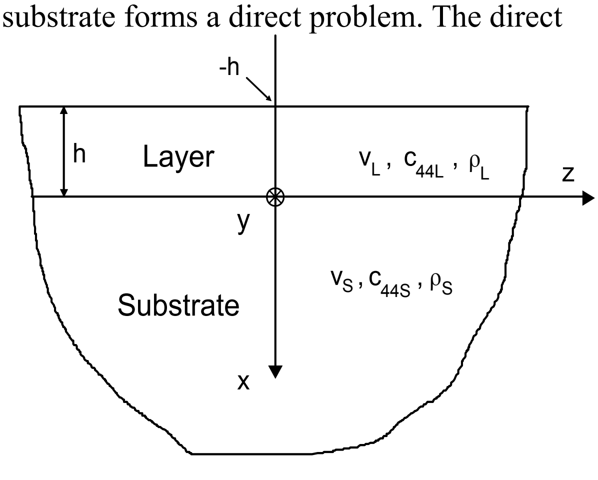

Fig.1. Geometry of a Love wave waveguide ($v_L < v_S$).

problem (direct Sturm-Liouville problem) describes the propagation of the Love wave in the layered media. The Love wave propagates in a semi-infinite layered structure shown in Fig.1. Here, an elastic isotropic layer is rigidly attached to an isotropic and elastic half-space. Mechanical vibrations of the shear horizontal surface wave are performed along the y axis parallel to the surface. The Love wave propagates along the z direction. The thickness of the layer is h.

Using the appropriate boundary conditions, we arrive at the following dispersion equation of the Love wave propagating in a layered half-space [4]:

$$\Omega = tan\left\{\sqrt{\frac{v^2\cdot\rho_L}{c_{44L}}-1}\cdot\frac{\omega\cdot h}{v}\right\}-\frac{c_{44S}}{c_{44L}}\frac{\sqrt{1-\frac{v^2\cdot\rho_S}{c_{44S}}}}{\sqrt{\frac{v^2\cdot\rho_L}{c_{44L}}-1}}=0 \qquad (1)$$

where:
h is the thickness of the surface layer
c_{44L} is the shear elastic constant of the layer
ρ_L is the density of the layer
ω is the angular frequency
v is the phase velocity of the Love wave
c_{44S} is the shear elastic constant of the substrate
ρ_S is the density of the substrate

It can be shown from Eq.1, that the phase velocity of the Love wave depends on the elastic properties of the layered structure, thickness and frequency. Solution of the dispersion equation (1) results in a series of discrete values of the Love wave velocity v_i, for a given value of frequency. Once the wave velocity v_i is known, the corresponding distribution $f_i(x)$ of the wave amplitude with depth x can be calculated. A set of pairs $\{v_i, f_i(x)\}$, where v_i is the surface wave velocity, and $f_i(x)$ the distribution of the wave amplitude with depth, constitutes the solution of the direct problem. The index $i=1$ refers to the fundamental mode. Higher modes of Love waves are labelled by $i>1$. In the present paper, we have restricted our attention to the propagation of the fundamental mode of Love waves.

III. INVERSE PROBLEM

The inverse problem relies on the determination of unknown material parameters from the measured dispersion curves of Love waves propagating in the considered layered structure. In this paper, the inverse problem was formulated and solved as an optimization problem [5] with properly defined objective function.

The objective function is a measure of the distance between the mathematical model of the investigated object and the real object. The objective function depending on the material parameters of the structure, frequency, and experimental data (phase velocity of the surface Love wave) was introduced and defined as:

$$\Pi = \sum_{j=1}^{N_e} \left| \Omega\left(h,\ c_{44L},\ \rho_L,\ \omega_j,\ v_j,\ c_{44S},\ \rho_S\right) \right| \quad (2)$$

where:

N_e - is the number of experimental points

ω_j – is the measured angular frequency

v_j - is the measured phase velocity

h - is a guess thickness of the layer

c_{44L} - is a guess elastic constant of the coating layer

ρ_L - is a guess density of the surface layer

c_{44S} - is the shear elastic constant of the substrate (known “a priori”)

ρ_S - is the density of the substrate (known “a priori”)

The design variables h, c_{44L}, ρ_L are arguments of the objective function Π and the quantities ω_j, v_j and c_{44S}, ρ_S in Eq.2 are parameters. Optimized are material parameters of the layer h, c_{44L}, ρ_L, whereas the material parameters of the substrate c_{44S}, ρ_S are given “a priori”.

Making use of the optimization methods a minimum of the objective function was determined. This enabled the determination of the optimum values for the unknown mechanical and geometrical parameters such as the elastic coefficient c_{44L} and thickness h of the thin coating layer. To minimize the considered objective function Π the appropriate optimization procedures from Mathcad® computer program were employed.

IV. DETERMINATION OF THIN LAYERS PARAMETERS

We solved the inverse problem for three cases. In case 1 only thickness h is unknown. In case 2 we assume that the thickness h and shear elastic constant c_{44L} are unknown, and in case 3, three parameters, i.e., the thickness h, shear elastic constant c_{44L}, and density ρ_L are not known. For example:

1) Inversion of thickness h of Cu layer:

Initial value: h = 0 m, constraints: 0 < h < 2e-3 m.

Results from the inverse method: h = 541 μm.

2) Inversion of thickness h and c_{44L} of Cu layer:

Initial values: h = 1e-4 m, c_{44L} = 3e+10 N m^{-2}, constraints: 0 < h < 2e-3 m, 3e+10 < c_{44L} < 5e+10 N m^{-2}.

Results from the inverse method: h = 473 μm, and c_{44L} = 3.76e+10 N m^{-2}.

3) Inversion of thickness h, c_{44L} and ρ_L of Cu layer:

Initial values: h = 1e-3 m, c_{44L} = 2e+10 N m^{-2}, ρ_L = 8e+3 kg m^{-3}, constraints: 0 < h < 2e-3 m, 3e+10 < c_{44L} < 5e+10 N m^{-2}, 7e+3 < ρ_L < 9e+3 kg m^{-3}.

Results from the inverse method: h = 486 μm, c_{44L} = 3.83e+10 N m^{-2}, and ρ_L = 9e+3 kg m^{-3}.

V. VERIFICATION

The exact values of the material parameters of the copper layer and steel substrate (see Table I) were determined from the geometrical and ultrasonic measurements. The thickness was measured using metallographic microscope, and the velocity of bulk shear acoustic waves was measured in the copper layer and steel substrate respectively. The density of copper and steel is known from physical tables.

Table I.

Exact material properties (thickness h, shear modulus c_{44} and density ρ) of the investigated layered structure Cu on steel.

Material	h	c_{44} [N m^{-2}]	ρ [kg m^{-3}]
Cu (copper) (layer)	500 μm	3.93e+10	8.9e+3
Steel (substrate)	10 mm	7.99e+10	7.8e+3

We compared the experimental dispersion curve to that obtained from the direct problem and calculated for the value of the thickness h = 541 μm, see Fig.2. Very good conformity of the theoretical and experimental dispersion curves has been stated.

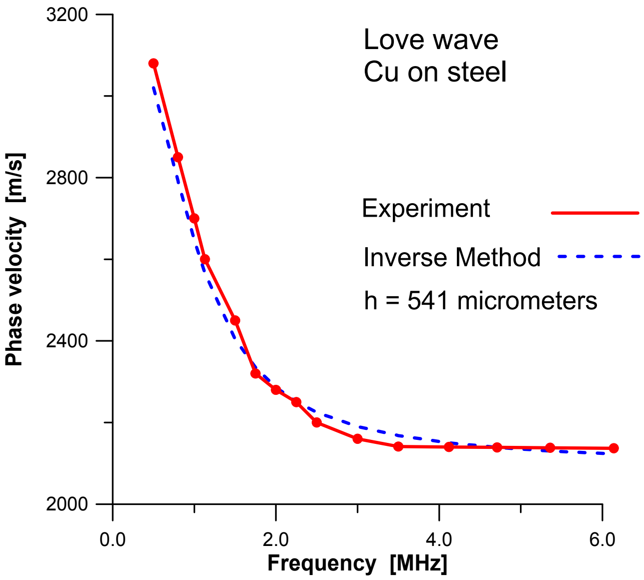

Fig.2. Comparison of the experimental dispersion curve with that obtained from the inverse method (Cu + steel structure).

VI. CONCLUSIONS

A new inverse method employing Love waves to extract the elastic and geometrical properties of thin layers from the measured dispersion relations was established. To the authors' knowledge, the application of Love waves for determining the mechanical properties of thin coating layers is a novelty. The usefulness of the ultrasonic method employing Love waves to investigate the elastic and geometrical properties of thin coating layers has been stated.

Employing shear surface waves (i.e., Love waves) for testing thin coating layers is more convenient than Rayleigh waves because the velocity of the Love wave depends upon only one elastic constant. This simplifies significantly the solution of the direct and inverse problem.

The direct problem was formulated and solved analytically. Theoretical dispersion curves were obtained.

The inverse problem was formulated as an optimization problem. Consequently, the objective function based on the dispersion equation was determined and minimized.

The obtained from the inverse method elastic and geometrical parameters were used as input data in the calculations of the direct problem. Resulting from the direct problem dispersion curves were compared with those measured experimentally. Good conformity between theoretical and experimental dispersion curves has been stated. This can evidence for the validity of the inverse method used for determining the mechanical properties of thin coating layers by means of shear horizontal surface waves of the Love type.

The presented measuring method and theoretical analysis can be also extended to the identification of the mechanical properties of other classes of modern materials such as composites, graded materials with continuous profiles, intermetallics etc.

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